Prototype Comprehensive Exam Questions
Accompanying Week 1

1. Consider that a random variable, S follows the following stochastic process:

$$S = \mu S dt + \sigma S dz$$

Sketch the path that S is expected to follow. Sketch a path that S may follow.

Use Itô's Lemma to show the process followed by $X = \ln S$. What is the transition density of S? What is the transition density of X?

2. Consider that S follows the following stochastic process:

$$S = \mu S dt + \sigma S dz$$

Solve for the following:

$$E\left[\max\left(S - X, 0\right)\right]$$

3. State the put-call parity relationship, and show how an arbitrageur may profit if it is violated.

4. Demonstrate how to obtain the sde for a call option, following Black and Scholes. Discuss the intuition behind this expression.

5. Discuss the veracity and usefulness of the following statement.

> In the context of using the absence of arbitrage to value a European call option on stock S, equivalent martingale pricing means that the risk premium S plays no role in valuing the option.

6. Discuss the veracity and usefulness of the following statement.

> Under the assumptions of the Black and Scholes model options are redundant assets. Therefore, the fact that options exist is sufficient justification to reject the model.

7. Derive the following under the Black and Scholes model for European calls and puts: delta, gamma, theta, vega, and rho. Show the sign of each for an at-the-money option that expires in t years. Explain what happens to each if we move deeper in-the-money, and as t increases.