Current draft: January 17, 2024
First draft: February 1, 2012

An Empirical Assessment of Characteristics and Optimal Portfolios

Christopher G. Lamoureux*
and
Huacheng Zhang†

Abstract

We implement a dynamically regularized, bootstrapped two-stage out-of-sample parametric portfolio policy to evaluate characteristics' efficacy in the conditional stock return generating process in the metric of expected power utility. Traditional characteristics, such as momentum and size afforded large utility gains before 1999. These opportunities have since vanished. Overfitting–imprecision in weight estimation–is correlated with the optimal portfolio's variance. Therefore, it is not a problem for power utility investors with coefficients of relative aversion greater than four. For more risk-tolerant investors, we successfully reduce estimation error by increasing the curvature of the loss function relative to the investor's utility function.

Key Words: cross-section of stock returns; overfitting; stock characteristics; optimal portfolios; out-of-sample evaluation

*Department of Finance, The University of Arizona, Eller College of Management, Tucson, 85721, 520–621–7488, lamoureu@arizona.edu.

†Department of Accounting and Finance, University of Edinburgh Business School, UK, zhanghuacheng1@gmail.com.

We are grateful to the editor, Clemens Sialm and two anonymous referees for helping us to improve our paper. We retain responsibility for all errors.

1. Introduction

Much of the empirical research in asset pricing over the past forty years examines the predictive content of measurable stock characteristics. We are interested in the question of whether risk-averse investors who care about all moments of the return distribution can optimally exploit this predictability. Furthermore, if the original findings of predictability are robust and economically significant for such an investor, have they vanished in recent years, as investors have learned of the predictability and their capabilities of exploiting that predictability have increased?

We answer these questions in the metric of power utility functions with a bootstrapped out of sample approach. We use Brandt, Santa-Clara, and Valkanov's (2009) parametric portfolio policy (PPP) to build characteristic-based portfolios that maximize in-sample power utility. We evaluate these portfolios out of sample. We confront estimation risk and model selection with a two-stage dynamically regularized out-of-sample design. We use the first out-of-sample stage to construct density functions of all of the optimal portfolios' certainty equivalent returns. We use a minmax procedure to select the optimal portfolio policy. We evaluate this optimal portfolio policy in a second out-of-sample stage. Since both the in- and out-of-sample periods are bootstrapped we construct (small sample) empirical distributions and report confidence intervals for functions of interest, such as portfolio alpha, Sharpe ratio, and certainty equivalent return.[1] Evaluating characteristics' predictive efficacy with this loss function, in the expected utility metric, where we consider all moments of portfolios' return distributions, addresses concerns about the statistical robustness and economic relevance of the return predictability. Furthermore, expected return predictability may persist in equilibrium if it is subject to large outliers and/or negative skewness. Barroso and Santa-Clara (2015a) and Kadan and Liu (2014) show that characteristic-based portfolios that generate a high alpha and Sharpe ratio may come at the cost of negative skewness. Nagel (2021, p. 33) notes, "whether methods that deliver the most accurate return forecasts at the individual stock level also automatically give us the best performing portfolio once we aggregate across stocks is an open question that does not have an obvious answer."

We find that over the period 1955-1998, all six of the characteristics that we consider: size, the book-to-market ratio, momentum, average same-month return, residual volatility, and beta have economically meaningful predictive content for the purpose of forming optimal portfolios from a power utility investor's perspective. Our primary performance metric is the portfolio's certainty equivalent return to a power utility investor with coefficient of relative risk aversion, $\gamma = 2$. In the out-of-sample period, 1990 - 1998, this investor's regularized dynamically optimized optimal portfolio's certainty equivalent return has a 95% confidence interval of (329 , 529) basis

[1] Lewellen, Nagel, and Shanken (2010) stress the importance of presenting confidence intervals since sample statistics in asset pricing are often biased and skewed.

points per month compared to the market portfolio's (121 , 137).

We analyze overfitting in Brandt, Santa-Clara, and Valkanov's (2009) PPP which has been used successfully in a variety of applications.[2] PPP is parsimonious and avoids the first step in traditional portfolio selection – estimating, or even taking a stand, on the conditional distribution of returns–given measurable characteristics. Aït-Sahalia and Brandt (2001, p. 1299) characterize this first step as the "Achilles' heel of conditional portfolio choice because although the moments are predictable, this predictability is for some moments quite tenuous." They argue that not specifying a likelihood (i.e., the conditional return distribution) avoids "introducing additional noise and potential misspecifications through the intermediate, but unnecessary, estimation of the return distribution."

Best and Grauer (1991) stress that overfitting causes the documented poor out-of-sample performance of optimal mean-variance portfolios. Optimization amplifies estimation errors. The literature suggests three approaches to mitigate overfitting in portfolio optimization settings. These regularization procedures include: constraining the weights or estimators, Bayesian priors, and machine learning. We use machine learning to manage estimation risk. Jagannathan and Ma (2003) demonstrate that there is a duality between constraining the weights, for example with short-selling constraints, and shrinking moment estimators in the mean-variance optimization context. Bayesian approaches establish a prior using economic theory. Pástor (2000) and Pástor and Stambaugh (2000) use asset pricing models to form the prior. MacKinlay and Pástor (2000) impose moment restrictions according to a factor model. Kan and Zhou (2007) derive the expected loss function from using sample (rather than true) moments when returns are normally distributed. They show that estimation risk can be diversified by holding a minimum variance portfolio in addition to the estimated tangency portfolio. These solutions which effectively reduce portfolio variance relative to the population solution suggest an approach to overfitting more generally by increasing the shadow cost of the return variance (and kurtosis) in terms of mean return (and skewness). DeMiguel, Garlappi, and Uppal (2009) show that in general these attempts to mitigate estimation error in (mean-variance) portfolio selection are dominated by an equally-weighted benchmark (the $\frac{1}{N}$ rule). Barroso, Reicheneker, Reicheneker, and Rouxelin (2023) consider benchmark constraints which limit the deviation of weights from standard equally-weighted and value-weighted benchmark portfolios. Since such constraints tend to reduce portfolio variance they mitigate estimation risk.

Machine learning approaches such as Lasso introduce a hyperparameter, or tuning parameter,

[2]DeMiguel, Plyakha, Uppal, and Vilkov (2013) use PPP to examine the predictive content of option-implied moments in a mean-variance setting. Faias and Santa-Clara (2017) analyze optimal option portfolios. Kroencke, Schindler, and Schrimpf (2014) and Barroso and Santa-Clara (2015b) consider foreign exchange portfolio strategies, including the carry trade. Barroso, Reicheneker, Reicheneker, and Rouxelin (2023) optimize jointly over global equity and currency exposure.

to manage estimation risk. For example, DeMiguel, Martín-Utrera, Nogales, and Uppal (2020) use a PPP algorithm to maximize the Sharpe ratio (i.e., they specify a quadratic utility function), with transactions costs. They impose an L1-norm penalty on the parameter space, and demonstrate that it does better out of sample than a non-regularized optimization. Freyberger, Neuhirl, and Weber (2020) use a group Lasso procedure to shrink the model and manage overfitting. Ao, Li, and Zheng (2019) develop a Lasso-type estimator to deal with a large cross-section specifically designed to address the out-of-sample deterioration of the Sharpe ratio. However, Kozak, Nagel, and Santosh (2020, p.274) note that such a penalty has poor statistical properties when the characteristics are correlated, and it lacks economic motivation. We regularize the PPP by separating the curvature of the loss function that links portfolio weights directly to characteristics from the investor's utility function. Under a power utility function, the coefficient of relative risk aversion, γ is effectively the shadow cost of variance relative to expected returns. We expand the parameter space to allow a power utility investor with coefficient of relative risk aversion γ to increase this shadow cost in sample by maximizing expected utility with coefficient of relative aversion $\gamma^* = \gamma + \lambda$. The hyperparameter $\lambda > 0$ will reduce estimation risk, if present, to the extent that noise is positively linked to the variance of the conditional return generating process.

We find that for mid-levels of relative risk aversion PPP does not suffer from estimation risk, lending credence to the claim that estimating moments of the conditional return distribution is the source of much overfitting (Aït-Sahalia and Brandt 2001). However, estimation risk is a serious problem for PPP to our power utility investor with a coefficient of relative risk aversion of two. Since the PPP is agnostic with respect to the conditional return generating process, we cannot appeal to Bayes' Theorem to manage estimation risk. Instead, we rely on the multiprior decision theory of Gilboa and Schmeidler (1989). Gilboa and Schmeidler (1989 p. 142) consider uncertainty–as distinct from risk– where "there is too little information to form a prior." They show that uncertainty aversion means that the agent should optimize over all feasible states and choose that rule which produces the best outcome under the worst possible state of nature.

Because there is no likelihood we use the bootstrap to construct the sampling distribution of out-of-sample portfolio properties for each model configuration. A configuration consists of the curvature of the loss function used to estimate an optimal portfolio rule in-sample (λ) and the (sub)set of measurable characteristics. With 6 characteristics there are $63 = \sum_{j=1}^{6} \frac{6!}{(6-j)! \cdot j!}$ unique combinations. We consider 14 values of λ. So we evaluate all $63 \times 14 = 882$ alternative configurations at the beginning of each year in our out-of-sample periods. After (minimally) 15 years of out of sample data we evaluate the utility function of these out-of-sample returns. We now have a bootstrapped sampling distribution of the utility function of the out-of-sample returns from each configuration. Confronted with a finite sample, the investor seeks to maximize expected utility in the worst-case scenario (i.e., maxmin). We select that configuration with the highest 1%ile value of the loss function (certainty equivalent return) at the beginning of each year in the second-stage out-of-sample period to construct the bootstrap distribution of the returns on the dynamically optimal portfolio policy.[3] This is linked to statistical assessment of the portfolio. We consider that Portfolio A dominates (i.e., is statistically significantly strictly preferred to) Portfolio B if: a) A's 2.5%ile certainty equivalent return is greater than B's 97.5%ile certainty equivalent return; and/or b) A's 2.5%ile certainty equivalent return is positive but B's 2.5%ile certainty equivalent return is negative.

Our use of the maxmin criterion on the bootstrapped out-of-sample certainty equivalent returns relates to the literature in decision making under uncertainty with machine learning. We learn from specifications' out-of-sample performance, as in Barroso and Saxena (2022) and Freyberger, Neuhierl, and Weber (2020). Gilboa, Postlewaite, and Schmeidler (2008) provide an overview and survey of the problem of decision making under uncertainty, and the "multiple prior" approach. Aït-Sahalia and Brandt (2001) suggest the use of maxmin for a CRRA investor in the case where the (conditional) return distribution is unknown. This approach has been extended broadly to dynamic optimization by Hansen and Sargent (2008). There is related work in operations research and machine learning on *robust optimization*, where "it is assumed that the decision maker has no distributional knowledge about the underlying uncertainty except for its support, and the model minimizes the worst-case cost over an uncertainty set," (Rahimian and Mehrotra 2019, p. 1). Bertsimas, Gupta, and Kallus (2018) characterize this approach as "data-driven robust optimization."

Nagel (2021, p. 48) notes that, "shrinkage can improve portfolio performance if there is heterogeneity in the covariates' relative contribution to moments and estimation error. Shrinkage must reduce undesirable contributions (estimation error, variance, and kurtosis) more than desirable ones (return mean and skewness)." We regularize or shrink estimation by disentangling the loss function maximized on the data to obtain portfolio weights from the investor's utility function. Our two findings, that $\lambda > 0$ greatly reduces estimation risk for our primary (relatively risk-tolerant) investor and $\lambda = 0$ for more risk-averse investors, suggest that estimation risk, the tendency to find spurious patterns in a sample, is related to the variance of the portfolio return distribution. This is fully consistent with all of the literature that demonstrates that constraining portfolio leverage (hence variance) serves to mitigate estimation risk.

We find that characteristics' predictive usefulness for portfolio selection is temporally unstable, and has vanished post-1998. A jump in λ under the rolling protocol prior to 2001 accompanies this

[3]We use the bootstrap 1%ile as the "worst-case scenario" to accommodate numerical issues and link to statistics. Our results do not change in any qualitative way if we use the 2.5%ile instead of the 1%ile certainty equivalent to select the optimal out-of-sample configuration.

structural break. Our minmax regularization suggests more conservative portfolios following out-of-sample results that are disappointing compared to prior expectations. This result is consistent with recent research. For example, Martin and Nagel (2022) provide a learning framework in which an econometrician can detect such *cross-sectional* predictability using standard tests, but the predictability is no longer present out of sample. They motivate the use of an out-of-sample test design–noting that we should expect to find evidence of predictability *in sample* in a high dimensional highly complex environment. In this setting, the usual implications of informationally efficient markets place testable restrictions on *out-of-sample* predictability. Green, Hand, and Zhang (2017) document a significant drop in characteristics' predictive content in 2003, which they attribute to institutional changes that reduce trading frictions.[4] McLean and Pontiff (2016) also document a drop in the return to trading on anomalies documented in the literature subsequent to publication. Our result, that measurable characteristics did have economically and statistically predictive content for portfolio construction prior to 1999, but no longer do, complements this research.

We consider power utility investors with higher aversions to risk, γ values of 5 and 8. The corresponding tables and figures are provided in an Internet Appendix. We confirm the findings that CRRA investors could use PPP to exploit the predictive content in characteristics prior to 1999, but not since then. As noted, while overfitting is a severe problem for the more risk-tolerant ($\gamma = 2$) investor in this first subperiod it is not for the more risk-averse investors. In the first subperiod optimal portfolio variance declines statistically significantly in risk aversion. These results are consistent with the hypothesis that estimation risk shrinks in portfolio variance. However, optimal portfolio Sharpe ratio, skewness, and kurtosis are flat in risk aversion.

We complement studies by Lewellen (2015), Green, Hand, and Zhang (2017), and Freyberger, Neuhirl, and Weber (2020) by showing *how* the set of characteristics affects optimal portfolios' factor exposures in this first subperiod. Our most risk-tolerant investor uses the characteristics to short the market, and get positive exposures to the Fama-French value, size, and momentum factors. Roughly half of the portfolio's excess mean return and return variance come from outside the span of the six Fama-French factors. The portfolio has a small positive exposure to their RMW factor, and a statistically significant negative loading on their CMA factor. While the size of characteristic weight tilts diminish in risk aversion, the percentages of portfolio mean returns and return variance within the span of the Fama-French six factor model are stable in γ.

[4]These changes stem from both regulations: Regulation FD (2000) and Sarbanes-Oxley (2002); and technological advances: decimalization (2001) and enhanced autoquote (2003).

2. Portfolio Selection

2.1 Algorithm

In Brandt, Santa-Clara, and Valkanov's (2009) algorithm, the vector θ is estimated to maximize a concave loss function over M periods:

$$\max_{\theta} \sum_{m=0}^{M-1} \frac{(1+r_{p,m+1})^{1-\gamma^*}}{1-\gamma^*} \left(\frac{1}{M}\right) \tag{1}$$

by allowing portfolio weights to depend on observable stock characteristics:

$$r_{p,m+1} = \sum_{i=1}^{N_m} \left(\overline{\omega}_{i,m} + \frac{1}{N_m}\theta' x_{i,m}\right) \cdot r_{i,m+1}\,, \tag{2}$$

where: $x_{i,m}$ is the K-vector of cross-sectionally standardized characteristics on firm i, measurable at month m; $\overline{\omega}_{i,m}$ is the weight of stock i in the (value-weighted) market portfolio at month m; and N_m is the number of stocks in the sample in month m. Conditioning only on information that is available to investors at the time the portfolios are formed avoids the overconditioning bias analyzed by Boguth, Carlson, Fisher, and Simutin (2011). Unlike Brandt, Santa-Clara, and Valkanov or DeMiguel, Martín-Utrera, Nogales, and Uppal (2020), we do not identify γ^*, the parameter used to generate a feasible portfolio strategy in (1) with the relevant statistical loss function (i.e., a specific investor's utility function). Instead, we consider a statistical loss function (alternatively "an investor") that takes the same form as (1) indexed by γ (the curvature of the statistical loss function which is pre-determined and fixed). From this perspective, γ^* is a choice variable – a tool to manage estimation risk. Letting $\gamma^* \equiv \gamma + \lambda$, λ is a hyperparameter, or tuning parameter of shrinkage or regularization.[5]

Our primary focus is on a power utility investor with coefficient of relative risk aversion, $\gamma = 2$. We also consider the effects of increasing risk aversion on estimation risk and the predictive content of characteristics for portfolio formation. Our interest is in out-of-sample statistical comparisons across portfolios generated by various feasible portfolio rules–from the perspective of this nonlinear statistical loss function. An eligible portfolio rule is a configuration consisting of a subset of characteristics and the increased penalty on variance and kurtosis, expressed in utility terms, λ. By definition, the optimal *in-sample* portfolio is obtained using all characteristics and by setting $\lambda = 0$, (i.e., $\gamma^* = \gamma$). Whether this is also true out of sample is an empirical question as it depends on the unmodeled relationship between estimation error, the characteristics and the loss function. If there is a positive relationship between an optimal portfolio's in-sample variance

[5] This algorithm is general and can be used with many alternative specifications. One extension that Brandt, Santa-Clara, and Valkanov (2009, Section 1.3.3) mention is allowing weights to be nonlinear functions of characteristics. Freyberger, Neuhirl, and Weber's (2020) finding that non-linear functions of characteristics improve the Sharpe ratio relative to a linear specification rationale for such an extension.

and its noise (i.e., higher *out-of-sample* variance) then using a more concave loss function than the investor's utility function ($\lambda > 0$), may generate portfolios that are preferred to those obtained by constraining γ^* to equal γ.

2.2 Data

Because our model selection stage uses out-of-sample analysis of expected utility from hundreds of configurations we require a comparatively long time series. Therefore we use characteristics that can be directly computed from market prices, and the book value from the CRSP-Compustat merged file. Our sample of returns and characteristics spans the years $Y_1 = 1960$ through $Y_{62} = 2021$, which means we use complete data starting in January 1955 to obtain all measurable characteristics as of the start of 1960. The initial in-period estimation uses the 180 months January, 1960 - December, 1974, and our initial out-of-sample validation period is 1975-1989. Our fully out-of-sample period comprises the 32 years 1990 – 2021. We use Y_y to indicate year Y, for $y = 1, \ldots,, 62$, and $m = 1, \ldots,, M = 744$ to denote the month in our sample.

For a stock to be eligible for investment in month m, we require five years of (non-missing) returns in months $[m-60,\ m-1]$. If the stock return is missing in month m, we look to the CRSP delisting return. If that is missing, we substitute -30% for NYSE– and AMEX–listed stocks and -50% for Nasdaq stocks. Thus the stocks in the January 1960 sample have no missing data from January 1955 through December 1959. These requirements limit the sample. For example, Brandt, Santa-Clara, and Valkanov (2009) report that the smallest cross-section in their study comprises 1,033 stocks in February 1964. Only 624 firms satisfy our data requirements in that month. Prior research suggests that it is important to exclude penny stocks and stocks with low relative market capitalization. To this end we add two additional criteria for a stock to be eligible for inclusion in month m. First, to exclude nano and small microcap stocks we impose a real dollar minimum equity market capitalization in month $m - 1$ of \$110 million in December 2021 dollars. We obtain the US Consumer Price Index from the Federal Reserve (FRED).[6] This restriction excludes stocks with market capitalization less than \$11.5 million in January 1960 and \$50 million in January 1990. Second, we exclude the smallest 10% of qualifying stocks in the months before Nasdaq stocks enter our sample, which is January 1978. We exclude 20% of eligible firms when Nasdaq stocks enter the sample. In February 1964, the dollar criterion discards 39 of the eligible 624 stocks, and the percent exclusion discards another 58 stocks. This leaves a final sample of 527 stocks – 51% of Brandt, Santa-Clara, and Valkanov's cross-section on that date.

Table IA-1 in the Internet Appendix provides details on the sample. For each of the 744 months in the full sample, the table shows: prior-month end, the number of stocks that meet the

[6]We extracted the series CPIAUCSL: consumer price index for all urban consumers: all items.

data availability requirement for month m (at month $m-1$), the number excluded by the minimum equity market capitalization constraint, and the final sample size; along with the minimum and median market capitalizations in the sample. There are 411 (exclusively New York Stock Exchange) stocks in the final sample in January 1960. The sample size jumps in August, 1967, from 678 to 881 when stocks listed on the American Stock Exchange are eligible for inclusion in our sample. The largest jump in sample size is in January 1978 (from 1,001 to 1,420 stocks) when Nasdaq stocks are eligible to enter our sample. The maximum number of stocks is 2,290 in April, 2006. There are 1,848 stocks in our sample in August 2008 and 1,693 stocks in the last month in our sample, December 2021.

We use the following six characteristics: momentum (ζ), the book-to-market ratio (V), log size (S), beta (β), market model residual standard deviation (σ_ϵ),, and the average same-month return over the preceding five years ($\overline{r}$).[7] Momentum is the stock's compounded return over the annual period $[m-13,\ m-2]$. Equity market capitalization is the market value of the company's outstanding shares (aggregated across all share classes) at time $m-2$. Book value is obtained from the Compustat database for the most recent fiscal year-end between $m-6$ and $m-18$. We define the book-to-market ratio to be the natural log of one plus the ratio of book value to equity market capitalization. Size is equity market capitalization. We estimate beta and the residual standard deviation by regressing monthly returns over the period $[m-60,\ m-1]$ on the CRSP value-weighted index.

We normalize and standardize the characteristics so that each characteristic has a cross-sectional mean of zero and unit standard deviation. Inspection of (2) shows that optimal portfolio weights will thereby sum to unity for any value of θ. This also means that the characteristics are observationally equivalent to shrinkage values. For example, let β be a stock's OLS beta. Consider a prior-weighted beta, such as $\beta^S = .5\cdot\beta + .5\cdot 1$. The normalized β^S are identical to the normalized β. This implicit shrinkage mitigates the usual concerns about outliers in characteristic space so we do not winsorize the characteristics. A single observation ($\Psi_{i,m}$) comprises stock i's return in month m, $r_{i,m}$, as well as the vector of characteristics, measurable at month $m-1$, for stock i, $i = 1, \ldots, N_m$. Importantly, $\Psi_{i,m}$ also includes stock i's market capitalization at $m-1$, since the passive portfolio at m (i.e., the portfolio when the θ-vector is zero) is the market-weighted portfolio at $m-1$.

We are interested in the optimal *sets* of characteristics so we consider all possible sets using these six characteristics. There are 63 such sets including each characteristic as a singleton and all six as a sextuplet.

[7] Same-month seasonality is analyzed in Heston and Sadka (2008).

2.3 Bootstrap sample construction

In light of the high noise-to-signal nature of stock return data, we develop and implement a bootstrap design for optimization, regularization and to characterize the out-of-sample sampling distributions of portfolio properties, such as certainty equivalent, portfolio loading on factors, portfolio skewness, etc. We stack the 744 row-vectors $\Psi_{i,m}$ (of varying lengths) to form the array $\mathbf{\Psi}$. This is not a traditional panel since stock i at month m is different from stock i at any other time, and as noted, the number of columns is different for each row.

Our resampling is nonparametric cross-sectional bootstrap (see Kapetanios 2008), motivated by the (repeated) single period optimization problem with out-of-sample cross-validation, regularization and inference. It is nonparametric since we draw from the raw data. It assumes that returns are temporally independent conditional on $\mathbf{\Psi}$. As such, $\mathbf{\Psi^{b}}$,is the b^{th} resample from $\mathbf{\Psi}$, for $b = 1, \ldots, B = 10,000$. Consistency holds under $N-$asymptotics (as the number of stocks in each period increases), since we can view $\hat{\theta}$ as a GMM estimator. Preserving the original time series in all resamples allows us to evaluate the effects of structural instability on out-of-sample portfolios, and also maintains the cyclical (month of the year) and serial dependence patterns inherent in the design of $\mathbf{\Psi_{m}}$, for $m = 1, \ldots, M = 744$. A resampled draw for month m resamples (with replacement) an N_m-row vector from $\mathbf{\Psi_{m}}$. Thus each pseudosample consists of the same number of observations in each period as the original sample and we preserve the calendar structure of the original data. We also maintain the temporal structure of the investment opportunity set. In the original data $\mathbf{\Psi_{m}}$ includes the raw values of the characteristics as well as the market weight of the stock at $m - 1$.

Once we draw a resampled cross-section from $\mathbf{\Psi_{m}}$ we normalize (so that all characteristics have a zero mean in this month in this bootstrapped sample), standardize (so that all characteristics have unit variances), and construct the value-weighted market weights for each resampled stock, based on the total market capitalization in this month in this bootstrapped sample. Thus our resampling design preserves the integrity of the investment opportunity set at each m. This critically includes the relationship between stocks' characteristics and sizes. We also maintain the integrity of the seasonality in the data. The month of January, for example occurs exactly once in any 12-month cycle, and its timing is deterministic. The matrix of characteristics at m is not independent of the size of each stock at m. We manage concerns about unmodeled temporal dependencies by maintaining the out-of-sample test design with each bootstrapped sample.

2.4 Dynamic regularization

We implement a dynamic two-stage bootstrapped out-of-sample regularization process using both updating and rolling construction. We provide an appendix containing pseudo code for our approach. Our first stage entails construction of the bootstrap distribution of out-of-sample returns from each of the 882 configurations. A configuration is a characteristic set and a value for λ (or γ^*). As noted above, with six characteristics, and 14 values of $\lambda = \{0,\ 1,\ \ldots,\ 11,\ 14,\ 20\}$, we consider 882 configurations under each protocol. In each of the $b = 1, \ldots, 10,000$ bootstrap samples $\boldsymbol{\Psi^b}$ we estimate the parameter vector $\hat{\theta}$ *in sample* by maximizing (1) over the $K-$vector θ for the in-sample period using a modified Newton method.[8] The first in-sample period (under both protocols) is 1960 - 1974. We use the $\hat{\theta}$ estimated from this period to form the out-of-sample portfolio in each of the next 12 months (in 1975). Under the updating protocol, then the 12 months of 1975 are added to the sample, and $\hat{\theta}$ is estimated on this 192-month in-sample period. This $\hat{\theta}$ is used to construct out-of-sample portfolio returns in the 12 months of 1976. Under the rolling protocol we drop the first 12 months from the in-sample data so that the second year's θ is estimated using the 180-month in-sample period 1961 - 1975. At this point we have 10,000 bootstrap draws of 24 months of out-of-sample returns from each of the 882 configurations.

Our dynamic regularization selects the optimal configuration by looking back at the performance of prior *out-of-sample returns*. We use at least 15 years of optimal portfolio out-of-sample returns for our second stage regularization, so the first year of second stage (i.e., dynamically regularized) out-of-sample returns is 1990. At the end of 1989 we have 180 months of 10,000 draws of out-of-sample optimal portfolio returns from each of the 882 configurations. We select that configuration whose bootstrap distribution of out-of-sample returns over the period 1975 - 1989 has the highest 1%ile certainty equivalent return as ex-post optimal, and use this to construct the dynamically optimized (and regularized) out-of-sample returns for the next 12 months (1990). From this point until the end of the sample each additional year involves re-estimating the parameter vector using the additional year's worth of data (as in Stage 1 optimization). Further, we (dynamically) regularize using the additional year of out-of-sample returns. We proceed in this manner through the end of 2020 to construct the dynamically optimized (and regularized) out-of-sample returns for 2021. The optimal configurations at the end of each year from both protocols are reported in Table 1, (and in Tables IA-2 and IA-3 for the dynamically optimized portfolios for the increasingly more risk-averse power utility investors). At any month in the second out-of-sample stage, the optimal configuration and its corresponding $\hat{\theta}$ vector are determined using information at the end of the previous year. The stocks' characteristics (and market weights) are available prior to the start of the month.

As in Brandt, Santa-Clara, and Valkanov (2009), we estimate the $\hat{\theta}$ coefficients at the beginning of each year in the out-of-sample period. There are several reasons for this. Since the

[8]We use the model/trust region algorithm of Gay (1983), and the analytical gradient and Hessian from (1). Our 2-stage algorithm is numerically intensive. We maximize in-sample utility 414,540,000 times.

updating protocol relies on the temporal structural stability of the return generating process, adding 12 months of data increases the reliability and efficiency of the θ estimates. By contrast, since the rolling protocol uses only the most recent 15 years of data, if the conditional return generating process experiences structural breaks, that will be evident in material changes in $\hat{\theta}$ over time. We similarly choose the optimal configuration using the minmax criterion at the beginning of each year in the second-stage out-of-sample period. The rolling protocol accommodates changes in the joint return, estimation error process. This dynamic regularization is consonant with the annual updating of $\hat{\theta}$, and is consistent with an investor updating her information set as she moves through time. In this design that new information is used in two ways. First to update the $\hat{\theta}$ vector, and second to select the optimal configuration. As the Appendix shows, the sampling protocols differ in both the in-sample period used to obtain $\hat{\theta}$ and the period used in second stage model selection.

3. Results

Table 1 shows the ex-post optimal portfolio policy for the power utility investor with $\gamma = 2$ at the beginning of each year in the second-stage out-of-sample period under both protocols. This is the optimal configuration from the first-out-of-sample period that ends before the indicated year: the characteristic set (of the 63 possibilities) and $\gamma^* = \gamma + \lambda$ (14 possibilities). The table provides sampling statistics (1%ile value, mean, and standard deviation) of this configuration's certainty equivalent from this period. Table 1 provides the same information about the sampling distribution of the two benchmarks' certainty equivalents at the beginning of the second-stage out-of-sample period. The benchmarks are the value-weighted portfolio of all eligible securities at the beginning of each month (VWI) and the equally-weighted portfolios of all eligible securities at the beginning of each month (EWI). The table also reports the certainty equivalent sampling distribution for the configuration with the optimal characteristic set and $\lambda = 0$ at the end of the first out-of-sample period under both protocols. Figure 1 reports the sampling distributions (box and whiskers plots) of the θ coefficient on each of the six characteristics for each year in the second-stage out-of-sample period from the rolling protocol. The whiskers show the 95% confidence interval on the estimate each year, and the box the interquartile range. The median is the bar inside the box.

Table 1 shows that $\lambda > 0$ is a necessary regularization for in-sample optimization to produce attractive out-of-sample returns for this investor under both protocols. This suggests that estimation risk is positively related to the variance of the conditional return distribution as $\lambda > 0$ penalizes variance (and kurtosis) more than the investor's utility function. This form of regularization has economic rationale–unlike penalizing the θ coefficients. It is also more statistically appealing since the characteristics' effects on portfolio returns are not independent, as noted by Kozak, Nagel, and Santosh (2020).[9]

Table 1 shows that this power utility investor's mean (95% confidence interval) out-of-sample certainty equivalent return without regularization (i.e., with $\lambda = 0$), over the 180 months ending in 1989 of -41% (-100% , 52%) per month under the updating protocol and -4% (-100% , 31%) per month under the rolling protocol. The power utility function is indeterminate at returns less than or equal to -100%. Therefore we define the certainty equivalent return for a pseudosample to be -100% (-10,000 basis points) per month if the return in any month in the relevant period does not exceed -1. The mean (95% confidence interval) of the θ coefficients in this case are: momentum, 5.8 (-30.5 , 10.8); book-to-market ratio, 5.3 (-2.7 , 9.5); log size, -16.5 (-37.5 , -10.9); residual volatility, -7.1 (-13.8 , 41.1); and average same-month return, 10.5 (-30.5 , 10.8). The enormous sampling variation in the weight coefficients shows the reason for the poor performance of this unregularized estimation. The asymmetry in the sampling distributions results from the correlatedness amongst the characteristics and cross-effects. This sampling variation in itself is enough to warn off investors from this configuration.

This type of estimation risk arises because of large absolute sampling correlations between the θ coefficients. In this case, the sampling correlations between the θ coefficient on residual volatility and the θ coefficients on momentum, average same-month return, and log size are -93%, -88%, and -86%, respectively. The sampling correlations between the θ coefficient on momentum and the θ coefficients on average same-month return and log size are 88% and 86%, respectively. As such, this type of overfitting cannot be solved by restricting some of the θ coefficients to zero (e.g., using an L1-norm penalty on θ). None of the 63 characteristic sets generates a portfolio which dominates the benchmark with $\lambda = 0$. By increasing the implicit penalty on the conditional return variance we are able to take advantage of the cross-effects amongst the characteristics.

Since the utility (statistical loss) function depends on higher moments, we report sampling distributions of measures of skewness and kurtosis. Our robust measures of skewness (S_4) and kurtosis (K_3) are recommended by Kim and White (2003):

$$S_4 = \frac{\mu - r_{.5}}{\sigma} \tag{3}$$

and

$$K_3 = \left\{ \frac{\overline{r}^{+}_{.95} - \overline{r}^{-}_{.05}}{\overline{r}^{+}_{.5} - \overline{r}^{-}_{.5}} - 2.63 \right\} \cdot 100 \tag{4}$$

[9] For example the θ coefficient on a characteristic may be centered close to 0 but takes on large positive values when the coefficients on other characteristics are small and takes on sizeable negative values when the coefficients on those other characteristics are large. This is especially important in light of the large sampling variation on all characteristics' θ estimates.

where: μ is the mean portfolio return over the out-of-sample period, σ is the return standard deviation, $\overline{r}^{+}_{.95}$ is the mean of the highest 5% of returns, $\overline{r}^{-}_{.05}$ is the mean of the smallest 5% of returns, $\overline{r}^{+}_{.5}$ is the mean of the top half of returns, $\overline{r}^{-}_{.5}$ is the mean of the bottom half of returns, and $r_{.5}$ is the median out-of-sample return.[10]

3.1 Out-of-sample regularization: Ex-post optimal configurations

The optimal configurations are very similar between the two protocols through the year 2000. Both specify $\lambda = 1$, and for the most part use all six characteristics. This similarity is somewhat surprising in light of the large sampling variation in the optimal portfolios' certainty equivalent returns. Many of the 882 alternative configurations have similar certainty equivalent distributions. For example, the optimal configuration under the updating portfolio for 1992 uses only the four characteristics: momentum, log size, average same-month return, and residual volatility, along with $\lambda = 1$. This configuration's 1%ile out-of-sample certainty equivalent return over the preceding 204 months is 512 basis points per month. The 1%ile of the configuration with $\lambda = 1$ and all six characteristics (i.e., add the book-to-market ratio and beta to the optimal set) is 509 basis points per month. The configuration with $\lambda = 2$ and all six characteristics has an analogous 1%ile of 452 basis points per month. Both characteristic sets with $\lambda = 0$ have 1%ile out-of-sample certainty equivalents of -100% per month.

Figure 1 shows the sampling distributions of the θ coefficients from the ex-post optimal portfolios under the rolling protocol preceding each year in the out-of-sample period. The figure illustrates the apparent structural change in the conditional return generating process around the year 2000. Prior to that point, the θ coefficients on all six characteristics tend to be large in absolute value, and their 95% confidence intervals are far from zero as well.

These optimal portfolios tend to tilt the portfolio most aggressively to low residual volatility stocks. They also tilt toward high beta stocks.[11] Prior to 2000, these ex-post optimal portfolios also tilt toward stocks whose average same-month return is high (in the next month) and away from those with relatively low average same-month return over the previous five years. The ex-post optimal portfolios also tilt toward stocks that have relatively high returns over months $[-13, -2]$, small stocks, and value stocks (those with relatively high book-to-market values).

The rolling protocol results in Table 1 show that the PPP's performance starts to deteriorate

[10] As per Kim and White (2003), in our pseudosamples S_4, the Pearson skewness coefficient is similar to, and more reliable than S_3, the Bowley skewness coefficient-integrated over the tail size. Similarly K_3, the Hogg coefficient, is more reliable than K_4, the Crow and Siddiqui parameter.

[11] The joint results on beta and residual volatility during this era are consistent with the findings in Liu, Stambaugh, and Yuan (2018). It is clear that prior to 1999, there is a relationship between variance and future expected returns. When the characteristic set contains both beta and residual volatility beta becomes desirable, and high residual volatility stocks appear undesirable, ceteris paribus When the characteristic set includes beta, but not residual volatility, then the sign on the beta θ coefficient becomes negative.

around 1999. Around this time the variance of the ex-post optimal portfolio's certainty equivalent return, with $\lambda = 1$, increases four-fold, and then λ increases to 5 and 6. Furthermore, the table shows that the optimal portfolio for the years 2010, 2011, 2013, 2014, 2015, and 2018, is the equally-weighted index of all sample stocks. This means that the 1%ile value of the certainty equivalent return over the preceding 15 years on all 882 configurations is less than this index's 1%ile certainty equivalent in that period, which is reported in the table. There is no evident breakdown under the updating protocol, as its ex-post optimal configurations are remarkably stable by comparison.

3.2 Optimal regularized portfolios' out-of-sample performance

3.2.1 Subperiod 1 (1990 - 1998)

Table 2 shows that both regularized updating and rolling protocols generate out-of-sample certainty equivalent returns that are statistically and economically significantly larger than the benchmarks in the first subperiod. The updating protocol produces a mean (95%ile confidence interval) certainty equivalent of 428 (329 , 529) basis points per month compared to the value-weighted index of the stocks in the universe of 129 (121 , 137). The optimal portfolios' Sharpe ratio means (95%ile confidence interval) under both protocols are 1.5 (1.3 , 1.8), significantly higher than the benchmark's 0.93 (0.86 , 1.00). The mean (95%ile confidence interval) certainty equivalent from the rolling protocol is: 497 (291 , 688) basis points per month. The optimal portfolios under both protocols have higher means and scales than both benchmark portfolios. Table 2 shows that both benchmarks are significantly negatively skewed: the 95%ile confidence interval on the value-weighted benchmark's S_4 (SKEW) is $(-11.6\,,\,-0.4)$. Both benchmarks also have significantly fatter tails than a Gaussian distribution. The 95%ile confidence interval on the value-weighted benchmark's K_3 (KURT) is (17 , 35). The dynamic optimal portfolios from both rolling and updating, in contrast, are symmetric and not significantly more leptokurtic than a Gaussian distribution. The rolling protocol generates a portfolio with higher scale and sampling variation, and lower minimum returns than the updating protocol.

The left-hand panel of Figure 2 contrasts the optimal dynamic portfolio from the updating protocol with the value-weighted benchmark. The difference in scale is apparent, as is the fact that the distribution of the optimal portfolio seems shifted to the right of the benchmark (higher mean and median returns).

Comparing characteristic-tilted portfolios and benchmarks in the metric of certainty equivalent returns makes no assumptions about the sources of systematic risk, or the factor structure of returns. Since the dynamic PPP dominates the benchmarks we next examine the relationship between these portfolios' returns and the Fama-French 6-factor model of returns. Table 3, Panel A provides information on *how* characteristics achieved higher utility in the pre-1999 era. This shows the projection of optimal portfolio returns minus the monthly riskless rate on the six Fama-French factors: the excess return on the value-weighted US stock market (Mkt), the value factor (HML), the (small) size factor (SMB), the momentum factor (MOM), the profitability factor (RMW), and the investment factor (CMA), obtained from Professor Kenneth French's website at Dartmouth. The loading on the market factor is significantly negative under both protocols. That is characteristics tilt the portfolio weights so that it is short the overall stock market factor. The power utility investor with $\gamma = 2$ instead seeks positive exposure to the value factor (HML), small stock factor (SMB), momentum factor (MOM), and the profitability factor (RMW). This portfolio has a significant negative loading on the conservative investment factor (CMA). The mean (95% confidence interval) portfolio alpha from the updating protocol is 264 (157 , 371) basis points per month. The analogous sampling statistics on alpha from the rolling protocol's optimal portfolio are 387 (210 , 578) basis points per month.

In this subperiod the characteristics shift the portfolio to lie outside the span of these six factors. Panel B of Table 3 decomposes the mean and variance of the updating protocol dynamic PPP's portfolio returns in excess of the risk free rate within the space of the 6-factor model. The orthogonal variance for each sample is the ratio of the residual variance from this regression to the portfolio variance. Variances attributed to the factors are the squared factor beta in the sample times the factor's variance. The variance values in the table do not sum to 100 in any bootstrap sample because the factors are not orthogonal. The largest pairwise correlations amongst the six factors in this first subperiod are: 77% between HML and CMA, -63% between the market and CMA, and -42% between SMB and RMW. On average (95% confidence interval), 53% (45% , 62%) of the variance in excess returns is not spanned by the 6-factor model. And 48% (33% , 61%) of the portfolio mean excess return comes in the form of alpha from this six-factor model. Within the factor span, momentum accounts for an average (95% confidence interval) of 48% (38% , 60%) of the optimal portfolio's expected excess return and 34% (25% , 43%) of its variance. HML is the third largest source of this portfolio's returns. HML accounts for 14% (11% , 19%) of the optimal portfolio's expected excess return and 38% (23% , 54%) of its variance. Table 3 Panel B shows that on average (95% confidence interval), 2% (0% , 4%) of the optimal portfolio's variance comes from exposure to the market factor.

3.2.2 Subperiod 2 (1999 - 2021)

Table 4 reports the properties of the dynamically optimized portfolios in the second-stage out-of-sample subperiod, 1999 - 2021. The right-hand panel in Figure 2 shows the return densities of the optimal portfolio from the updating protocol and the equally-weighted portfolio of sample stocks in this period. After 1998, both benchmark portfolios dominate the regularized dynamic parametric portfolio. The mean (95%ile confidence interval) certainty equivalent from the updating dynamic PPP is -2 $(-110\,,\,83)$ basis points per month, whereas these statistics are 77 $(73\,,\,80)$ basis points per month for the equally-weighted benchmark and 60 $(53\,,\,67)$ basis points per month for the value-weighted benchmark. The dynamic parametric portfolios under both protocols have significantly higher mean returns than the benchmarks. The Sharpe ratio from the updating protocol is not statistically different from the benchmarks' Sharpe ratios. However the equally weighted benchmark's Sharpe ratio is significantly larger than that generated by the rolling protocol's dynamic PPP.

As in the first subperiod, characteristic-tilts generate portfolios that are not negatively skewed– unlike the benchmarks. However, in this subperiod the dynamic PPP is dominated by the benchmarks for several reasons. First, characteristics are used to increase the scale of the distribution (measured by the interquartile range) by some 3.4 fold in both subperiods. The portfolio median return is only 1.9 times higher than the benchmark in this subperiod, whereas this ratio is 3.5 in the first subperiod. Another feature of the characteristic-based portfolios that changes from the first to the second subperiod is heightened kurtosis–manifest in the large negative returns. This is especially true for the rolling protocol under which at least one monthly return is less than -100% in more than 25% of the bootstrap samples, and less than -87% in more than 75% of these samples. Even though the power utility investor with $\gamma = 2$ is relatively risk tolerant, losing 100% results in a certainty equivalent return of that same magnitude.

Table 3, Panel C shows that the optimal characteristic-based portfolio from the updating protocol has a mean (95% confidence interval) alpha of 81 $(14\,,\,150)$ basis points per month, which is significantly positive. This highlights the possibility for discrepancies between the performance metrics, as this portfolio's Sharpe ratio is not significantly different from the benchmark. Because it results in a negative certainty equivalent return in more than 25% of the bootstrap samples, we consider it dominated in the metric of expected utility. This discrepancy is even more pronounced by the optimal dynamic parametric portfolio from the rolling protocol. This portfolio's alpha is larger (although not significantly so) than that from the updating protocol. Yet this rule produces a portfolio whose certainty equivalent is less than -21% (per month) in more than half of the bootstrap samples. Comparing the two periods, the biggest effect on the portfolio mean is the drop in the loading on momentum, whose mean drops from 2.6 to 1.0. The mean return on this factor dropped from 100 to 26 basis points per month.[12] The second largest effect is the drop in alpha–the mean alpha drops by 183 basis points per month.

[12] This phenomenon likely related to the momentum crash of 2009, documented by Barroso and Santa-Clara (2015a) and Daniel and Moskowitz (2016).

These effects are consistent with the properties of the θ coefficients under the rolling protocol, shown in Figure 1. The optimal portfolios shift toward the market factor in this period. The mean (95% confidence interval) loading on the market portfolio in this period is 0.9 (0.7 , 1.1) under the updating protocol and 1.3 (1.1 , 1.5) under the rolling protocol.

3.3 Updating versus rolling

In our empirical design we do not allow the investor to choose between rolling and updating, since the out-of-sample periods used for identifying the ex-post optimal configurations are not the same across the two protocols. Instead, as econometricians we compare results across protocols to make inferences about the nature of the data generating process.

Table 1 shows that the updating protocol does not provide any warning that characteristics' predictive value has vanished. Instead, the ex-post optimal model appears temporally stable. The large gains over the first 15 years of the configuration selection period disguise the fact that the PPPs underperform benchmarks in the more recent years. For example, were we to evaluate the model using the ex-post optimal portfolio ending in 2018 (Table 1), we would infer that the optimal value for λ is 1, and the optimal characteristic set comprises: momentum, log size, beta, average same-month return, and residual volatility. Indeed, over the out-of-sample period from 1975 - 2018 this portfolio dominates the benchmarks. However this masks the fact that this portfolio's certainty equivalent mean (95% confidence interval) over the preceding 180 months is -23 (-113 , 45) basis points per month. The salutary historical performance over the 44-year out-of-sample period trades off the positive effects of characteristic-based tilting in the years 1975 - 1998 against the negative effects in the latter years. This underscores the issues raised in Martin and Nagel (2022). The flexibility to choose the characteristic set and the hyperparameter, λ expands the dimensionality of the model space. The results of using an ex-post optimal model configuration (on out-of-sample returns) must be analyzed in a subsequent (truly) out-of-sample period to evaluate the model.

Table 1 suggests that some of the usefulness of λ as a regularization parameter may derive from its capacity to detect a structural break. However this is primarily true under the rolling protocol. Once there is a year in which the out-of-sample results deteriorate there is a jump in λ, as the algorithm scales back its reliance on characteristics. When 2000 replaces 1985 in the out-of-sample validation period λ jumps from 1 to 6. This increase in λ has a large impact on the out-of-sample certainty equivalent returns. For contrast, the 1%ile (95% confidence interval) on the certainty equivalent return from the configuration that was optimal in the preceding year, with $\lambda = 1$, is -10,000 ($-10,000$, 398.4).

Since Table 2 describes a noisy, structurally stable environment, the updating protocol produces portfolios with higher 2.5%ile certainty equivalents than rolling for all three investors. However the rolling protocol produces higher 97.5%ile certainty equivalents than updating. In a structurally stable environment more data is strictly preferred. Table 1 shows that the two protocols have very similar ex-post optimal configurations (λ and characteristic sets) through 1999.

3.4 Increasing risk aversion

Figures IA-1 - IA-6 show the confidence intervals of the ex-post optimal θ values from the rolling protocol preceding each year in the second-stage out-of-sample period, for each of the six characteristics for three power utility investors, with coefficients of relative risk-aversion: 2, 5, and 8. The values for the least risk-averse investor correspond to those reported in Figure 1. The figures show that as risk-aversion increases the optimal effect of characteristics on portfolio weights decreases in absolute value. Further, the apparent appeal of characteristic tilt appears to vanish more quickly for the more risk-averse investors. For example, the ex-post optimal out-of-sample portfolio put a significant positive weight on average same month return for the first 20 years in the second-stage out-of-sample period, for the most risk-tolerant investor–although there is a significant drop in the coefficient after 2000 (for the 2001 out-of-sample portfolio). However, this coefficient is zero for the most risk-averse investor in years 12 - 26 and 28 - 32.

Tables IA-2 and IA-3 provide the ex-post optimal configurations prior to each year in the second-stage out-of-sample period, from both protocols (analogous to Table 1) for the power utility investors with $\gamma = 5$ and $\gamma = 8$, respectively. Both power utility investors optimally set $\lambda = 0$ for most of the first subsample, confirming Brandt, Santa-Clara, and Valkanov's conjecture that their algorithm has much less estimation risk than mean-variance optimization. Contrasting this with the importance of $\lambda > 0$ for the most risk-tolerant of these three investors provides additional evidence that estimation risk (the tendency to overfit) is linked to the return variance. The power utility investor with $\gamma = 2$ is tolerant enough of variance (and kurtosis) so that maximizing this utility function directly on the data accepts positions with attractive in-sample expected return and skew that result from noise in the conditional return generating process. In this case using $\lambda = 1$ helps to separate the predictability from the noise.

These tables also demonstrate similar evidence from the rolling protocol, as in Table 1 that the fit of the model starts to deteriorate around 1999. The equally-weighted benchmark is the ex-post optimal portfolio (i.e., the 1%ile of its certainty equivalent sampling distribution is greater than all 882 configurations) in 2009 - 2015 for the power utility investor with $\gamma = 5$. Similarly the power utility investor chooses either the value-weighted benchmark or the equally-weighted benchmark above all 882 optimized portfolios prior to the years: 2002, 2005, and 2009 - 2015.

Table IA-4 is analogous to Table 2. It shows that in the first true out-of-sample period (1990 - 1998), the dynamic PPP portfolios under the updating protocol dominate the benchmark portfolios for both of these more risk-averse power utility investors. However, unlike the case for our most risk-tolerant investor, the optimal dynamic PPP portfolios from the rolling protocol do not dominate the benchmarks in this period for these investors. These results jointly lend credence to the hypothesis that the conditional return generating process was largely stable over the 1955 - 1998 period, as using more information reduces estimation risk. This table also shows that the optimal dynamic portfolios from the updating protocol are neither negatively skewed nor more leptokurtic than the Gaussian distribution.

The left-hand panels in Figures IA-7 and IA-8 contrast the return distributions of the optimal portfolio from the updating protocol with the preferred benchmark, which is the value-weighted benchmark in both cases in the first subperiod. Figure IA-8 is especially revealing as our most risk-averse investor's optimal portfolio has a similar scale as the benchmark. The mean (95% confidence interval) interquartile range of the optimal dynamic portfolio is 702 (576 , 840) basis points per month compared with the value-weighted benchmark's 470 (427 , 515) basis points per month. This figure's left panel shows that the optimal portfolio has more significant mass above a 5% monthly return than the benchmark, while having similar left-tail properties in the first subperiod. Both dynamic optimal portfolios have similar Sharpe ratios to that in Table 2 for our most risk-tolerant investor. Unlike the certainty equivalent, the Sharpe ratios from both protocols are very similar–both are significantly larger than the benchmarks'.

Table IA-5 reports the out-of-sample properties of the dynamic optimal portfolios for these two investors in our second-stage out-of-sample subperiod, 1999 - 2021. The mean certainty equivalent is negative for both investors' dynamic optimal portfolios under both protocols in this subperiod. The updating portfolios' Sharpe ratios are not significantly different from the benchmarks', however both investors' optimal dynamic portfolios' Sharpe ratios under the rolling protocol are significantly lower than the equally-weighted benchmarks'.

Table IA-6 contrasts the dynamic PPP with the benchmarks for the entire 384 month out-of-sample period (1990 - 2021). The optimal portfolios for for our focal investor (whose $\gamma = 2$) is the equally weighted benchmark, and the optimal portfolio for the two more risk-averse investors is the value weighted benchmark. As in the second subperiod, the dynamic PPP is less attractive in the expected utility metric because of its additional kurtosis. These portfolios from the updating protocol, for the two more risk-averse investors have a significantly higher Sharpe ratio than the benchmarks. Unlike the utility function, the Sharpe ratio does not over-weight the smaller minimum returns. The mean (95% confidence interval) of the value weighted benchmark's minimum monthly return over this period is -17% (-18% , -15%) per month. These statistics for our most risk-averse investor's dynamic PPP are -33% $(-44\%\,,\,-25\%)$ per month, from the updating protocol. These distributions are compared graphically in the right-hand panels of Figures IA-7 and IA-8. Both of these graphs show that the left tail of the dynamic PPP's return distribution dominates that of the preferred benchmark.

Tables IA-7 and IA-8 report the relationships between these two more risk-averse investors' optimal portfolios and the six Fama-French factors for all three out-of-sample periods, the first and second subperiods and the full 32-year period. Consistent with the analysis of these portfolios above, alpha decreases in γ. In the first subperiod our most risk-averse investor's optimal portfolio has mean (95% confidence interval) alpha of 104 $(60\,,\,149)$ basis points per month, (Table IA-8). Table IA-9 provides the sampling distributions of the power utility investor with $\gamma = 2$ dynamic optimal portfolio return projection on the Fama-French six factors for the full 32-year out-of-sample period. Table IA-10 reports the mean and variance decompositions for the two more risk-averse investors' optimal portfolios in the first subperiod (analogous to Table 3, Panel B). During this subperiod, when the characteristics provide useful information about future returns the variance decompositions are constant across risk aversion. For all three investors the six factors account for 40 to 55% (95% confidence intervals) of the optimal portfolio's return variance. Similarly, HML and MOM are the largest sources of return variance for all three portfolios amongst the six factors for all three investors.

In the period when characteristics were efficacious, the optimal portfolios' scales decrease in risk aversion. The interquartile range of returns for our most risk-tolerant investor has a mean (95% confidence interval) of 1,495 $(1,201\,,\,1,822)$ basis points per month; the analogous statistics for our most risk-averse investor's dynamic optimal portfolio are 702 $(576\,,\,840)$ basis points per month. However the sources of variance in the span of the six Fama-French factors are flat in γ. This result is fully consistent with the notion that sources of predictable excess return require exposure to non-diversifiable variance, as discussed by Kozak, Nagel, and Santosh (2020). Since all three investors tilt their optimal portfolios toward stocks that have done relatively well in the same month over the past five years, this result is consistent with Keloharju, Linnainmaa, and Nyberg's (2016) hypothesis that there are many small seasonal (month-of-the-year) factors, and return in the month serves as an instrument for exposure to these factors.

4. Conclusions

We explore the nature of estimation risk in conjunction with Brandt, Santa-Clara, and Valkanov's parametric portfolio policy. We use a bootstrapped out-of-sample maxmin criterion to select the optimal portfolio configuration at the beginning of each year in a second-stage (optimized) out-of-sample period. To gain insight into the interactions between portfolio optimization and estimation risk, we introduce a new form of regularization. Rather than penalizing the parameter space we introduce a hyperparameter that can increase the curvature of the loss function used to estimate portfolio weights. For power utility investors with moderate to high risk aversion (coefficients of relative risk aversion of 5 and 8), Brandt, Santa-Clara, and Valkanov's (2009) PPP algorithm afforded a way to exploit this predictability without significant estimation error. The most risk-tolerant power utility investor we consider, with coefficient of relative risk aversion of two, experiences much higher estimation risk with the parametric portfolio policy. This investor reduces overfitting by optimizing a power utility function with a higher coefficient of relative risk aversion than her own in sample. This loss function increases the shadow cost of variance in terms of expected return. It works because estimation error increases in portfolio variance.

Our results suggest that measurable characteristics did have economically and statistically significant predictive content for portfolio construction prior to the year 1999. That result considers all moments of the predictive distribution and is not specific to expected return, alpha, or Sharpe ratio. Since during this period (the 20th Century), optimal portfolios' characteristic-tilts diminished in risk aversion, we do not infer that any of the opportunities afforded by conditioning on characteristics represented a free lunch. Rather there were dimensions wherein the tradeoffs between mean (and skewness) and variance (and kurtosis) were more attractive than those afforded by the market portfolio. Two of those dimensions appear to be the well-known momentum and value factors. Optimal portfolio returns were virtually orthogonal to the market factor and one-half of their return variance came from outside the span of the Fama-French 6-factor model.

Characteristics' predictive efficacy for portfolio optimization has vanished starting in 1999. The market portfolio dominates all of the dynamically optimal parametric portfolios over the 1999 - 2021 period. This finding is consistent with the literature, which contains several non-mutually exclusive hypotheses. Martin and Nagel (2022) show that a complex economy in which agents learn about predictive relationships econometricians should expect to find in-sample predictability that vanishes out of sample. McLean and Pontiff (2016) suggest that investors adapt to academic research. Green, Hand, and Zhang (2017) note that the 21st Century has seen new regulations and technological advances that serve to reduce trading frictions. This, in turn, allows investors to more fully exploit predictive relationships.

Appendix

This appendix provides a pseudo code for our bootstrap dynamically regularized out-of-sample empirical design, with both updating and rolling sample protocols. Each year we optimize expected utility in sample to construct the next year's out-of-sample returns. Once we have at least 180 months of out-of-sample returns, we select the optimal configuration with numerical minmax of out-of-sample certainty equivalent returns. The optimal dynamically regularized policy–or second out-of-sample stage–has this configuration's out-of-sample returns in the next year. As in the text, we use the following notation: $y = 1, \ldots, 62$ references the number of each year in our sample. Uppercase Y is the year: $Y_1 = 1960$, $Y_{15} = 1974$, $Y_{30} = 1989$, and $Y_{62} = 2021$.

FOR each year, $y = 15, 16, \ldots, 29$:

FOR each configuration, $c = 1, 2, \ldots, 882$:

FOR each bootstrap sample, $b = 1, 2, \ldots, 10,000$:

Form out-of-sample returns: Maximize (1) over θ – using $[Y_s,\ Y_y]$.* These $\hat{\theta}_{y,c,b}$ are used to construct the out-of-sample portfolio returns in the year Y_{y+1}, for configuration c and bootstrap sample b, respectively.

END FOR

END FOR

END FOR

FOR each year, $y = 30, 31, \ldots, 61$:

Dynamic Regularization: Identify which of the 882 configurations' *out-of-sample portfolio returns* over years $[Y_v,\ Y_y]$ has the maximum 1%ile value certainty equivalent return.** This optimal configuration, c_y^*, is reported by year (Y_{y+1}) in Table 1 for the power utility investor with coefficient of relative risk aversion of 2. Tables IA-2 and IA-3 report this optimal configuration by year for power utility investors with coefficients of relative risk aversion of 5 and 8, respectively.

FOR each configuration, $c = 1, 2, \ldots, 882$:

FOR each bootstrap sample, $b = 1, 2, \ldots, 10{,}000$:

Form out-of-sample returns: Maximize (1) over θ – using $[Y_s,\ Y_y]$.* These $\hat{\theta}_{y,c,b}$ are used to construct the out-of-sample portfolio returns in the year Y_{y+1}, for configuration c and bootstrap sample b, respectively.

END FOR

END FOR

The bootstrap set of second stage, *out-of-sample* dynamically regularized optimal portfolio returns for the 12 months in year $y + 1$ is: $\{r_{y+1,c_y^*}\}$.

END FOR

*Under the updating protocol, $Y_s \equiv Y_1 = 1960$, and $Y_s = Y_{y-14}$ under the rolling protocol.

**Under the updating protocol, $Y_v \equiv Y_{16} = 1975$, and $Y_v = Y_{y-14}$ under the rolling protocol.

The replication code is available at https://doi.org/10.7910/DVN/LK4DCN.

References

Aït-Sahalia, Yacine and Michael W. Brandt, 2001, Variable selection for portfolio choice, *Journal of Finance* 56, 1297–1351.

Ao, Mengment, Yingying Li, and Xinghua Zheng, 2019, Approaching mean-variance efficiency for large portfolios, *Review of Financial Studies* 32, 2890–2919.

Barroso, Pedro, Jurij-Andrei Reichenecker, Michael Reichenecker, and Florent Rouxelin, 2023, An international equity and currency optimisation with frictions, Working paper, Catloica-Lisbon School of Business and Economics.

Barroso, Pedro and Pedro Santa-Clara, 2015a, Momentum has its moments, *Journal of Financial Economics* 116, 111–120.

Barroso, Pedro and Pedro Santa-Clara, 2015b, Beyond the carry trade: Optimal currency portfolios, *Journal of Financial and Quantitative Analysis*, 50, 1037–1056.

Barroso, Pedro and Konark Saxena, 2022, Lest we forget: Learn from out-of-sample forecast errors when optimizing portfolios, *Review of Financial Studies* 35, 1222-1278.

Bertsimas, Dimitris, Vishal Gupta, and Nathan Kallus, 2018, Data-driven robust optimization, *Mathematical Programming, Series A* 167, 235–292.

Best, Michael J. and Robert R. Grauer, 1991, On the sensitivity of mean-variance-efficient portfolios to changes in asset means: Some analytical and computational results, *Review of Financial Studies* 4, 315–342.

Boguth, Oliver, Murray Carlson, Adlai Fisher, and Mikhail Simutin, 2011, Conditional risk and performance evaluation: Volatility timing, overconditioning, and new estimates of momentum alphas, *Journal of Financial Economics* 102, 363–389.

Brandt, Michael W., Pedro Santa-Clara, and Rossen Valkanov, 2009, Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns, *Review of Financial Studies* 22, 3411–3447.

Daniel, Kent and Tobias J. Moskowitz, 2016, Momentum crashes, *Journal of Financial Economics* 122, 221–247.

DeMiguel, Victor, Lorenzo Garlappi, and Raman Uppal, 2009, Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?, *Review of Financial Studies* 22, 1915–1953.

DeMiguel, Victor, Alberto Martín-Utrera, Francisco J. Nogales, and Raman Uppal, 2020, A transaction-cost perspective on the multitude of firm characteristics, *Review of Financial Studies* 33, 2180–2222.

DeMiguel, Victor, Yuliya Plyakha, Raman Uppal, and Grigory Vilkov, 2013, Improving portfolio selection using option-implied volatility and skewness, *Journal of Financial and Quantitative Analysis* 48, 1813–1845.

Faias, José and Pedro Santa-Clara, 2017, Optimal option portfolio strategies: Deepening the puzzle of index option mispricing, *Journal of Financial and Quantitative Analysis* 52, 277–303.

Freyberger, Joachim, Andreas Neuhierl, and Michael Weber, 2020, Dissecting characteristics nonparametrically, *Review of Financial Studies* 33, 2326–2377.

Gay, David M., 1983, Subroutines for unconstrained minimization using a model/trust-region approach, *ACM Transactions on Mathematical Software* 9, 503–524.

Giglio, Stefano, Yuan Liao, and Dacheng Xiu, 2021, Thousands of alpha tests, *The Review of Financial Studies* 34, 3456–3496.

Gilboa, Itzhak, Andrew W. Postlewaite, and David Schmeidler, 2008, Probability and uncertainty in economic modeling, *Journal of Economic Perspectives* 22, 173–188.

Gilboa, Itzhak and David Schmeidler, 1989, Maxmin expected utility with non-unique prior, *Journal of Mathematical Economics* 18, 141–153.

Green, Jeremiah, John R.M. Hand, and X. Frank Zhang, 2017, The characteristics that provide independent information about average U.S. monthly stock returns, *Review of Financial Studies* 30, 4389–4436.

Hansen, Lars P. and Thomas J. Sargent, 2008, *Robustness*, Princeton University Press, Princeton, NJ.

Heston, Steve L. and Ronnie Sadka, 2008, Seasonality in the cross-section of stock returns, *Journal of Financial Economics* 87, 418–445.

Jagannathan, Ravi and Tongshu Ma, 2003, Risk reduction in large portfolios: Why imposing the wrong constraint helps, *Journal of Finance* 58, 1651–1683.

Kadan, Ohad and Fang Liu, 2014, Performance evaluation with high moments and disaster risk, *Journal of Financial Economics* 113, 131–155.

Kan, Raymond and Guofu Zhou, 2007, Optimal portfolio choice with parameter uncertainty, *Journal of Financial and Quantitative Analysis* 42, 621–656.

Kapetanios, George, 2008, A bootstrap procedure for panel data sets with many cross-sectional units, *The Econometrics Journal* 11, 377–395.

Keloharju, Matti, Juhani T. Linnainmaa, and Peter Nyberg, 2016, Return seasonalities, *Journal of Finance* 71, 1557–1589.

Kim, Tae-Hwan and Halbert White, 2003, On more robust estimation of skewness and kurtosis: Simulation and application to the S&P 500 Index, Working Paper, University of California, San Diego.

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh, 2020, Shrinking the cross section, *Journal of Financial Economics* 135, 271–292.

Kroencke, Tim, Felix Schindler, and Andreas Schrimpf, 2014, International diversification benefits with foreign investment styles, *Review of Finance* 18, 1847–1883.

Lewellen, Jonathan, 2015, The cross section of expected stock returns, *Critical Finance Review* 4, 1–44.

Lewellen, Jonathan, Stefan Nagel, and Jay Shanken, 2010, A skeptical appraisal of asset pricing tests, *Journal of Financial Economics* 96, 175–194.

Liu, Jianan, Robert F. Stambaugh, and Yu Yuan, 2018, Absolving beta of volatility's effects, *Journal of Financial Economics* 128, 1–15.

MacKinlay, A. Craig and Luboš Pástor, 2000, Asset pricing models: Implications for expected returns and portfolio selection, *Review of Financial Studies* 13, 883–916.

Martin, Ian and Stefan Nagel, 2022, Market efficiency in the age of big data, *Journal of Financial Economics* 145, 154–177.

McLean, R. David and Jeffrey Pontiff, 2016, Does academic research destroy stock return predictability? *Journal of Finance* 71, 5–31.

Nagel, Stefan, 2021, *Machine Learning in Asset Pricing*, Princeton University Press.

Pástor, Luboš, 2000, Portfolio selection and asset pricing models, *Journal of Finance* 60, 179–223.

Pástor, Luboš and Robert F. Stambaugh, 2000, Comparing asset pricing models: An investment perspective, *Journal of Financial Economics* 56, 335–381.

Rahimian, Hamed and Sanjay Mehrotra, 2019, Distributionally robust optimization: A review, Working paper, Northwestern University.

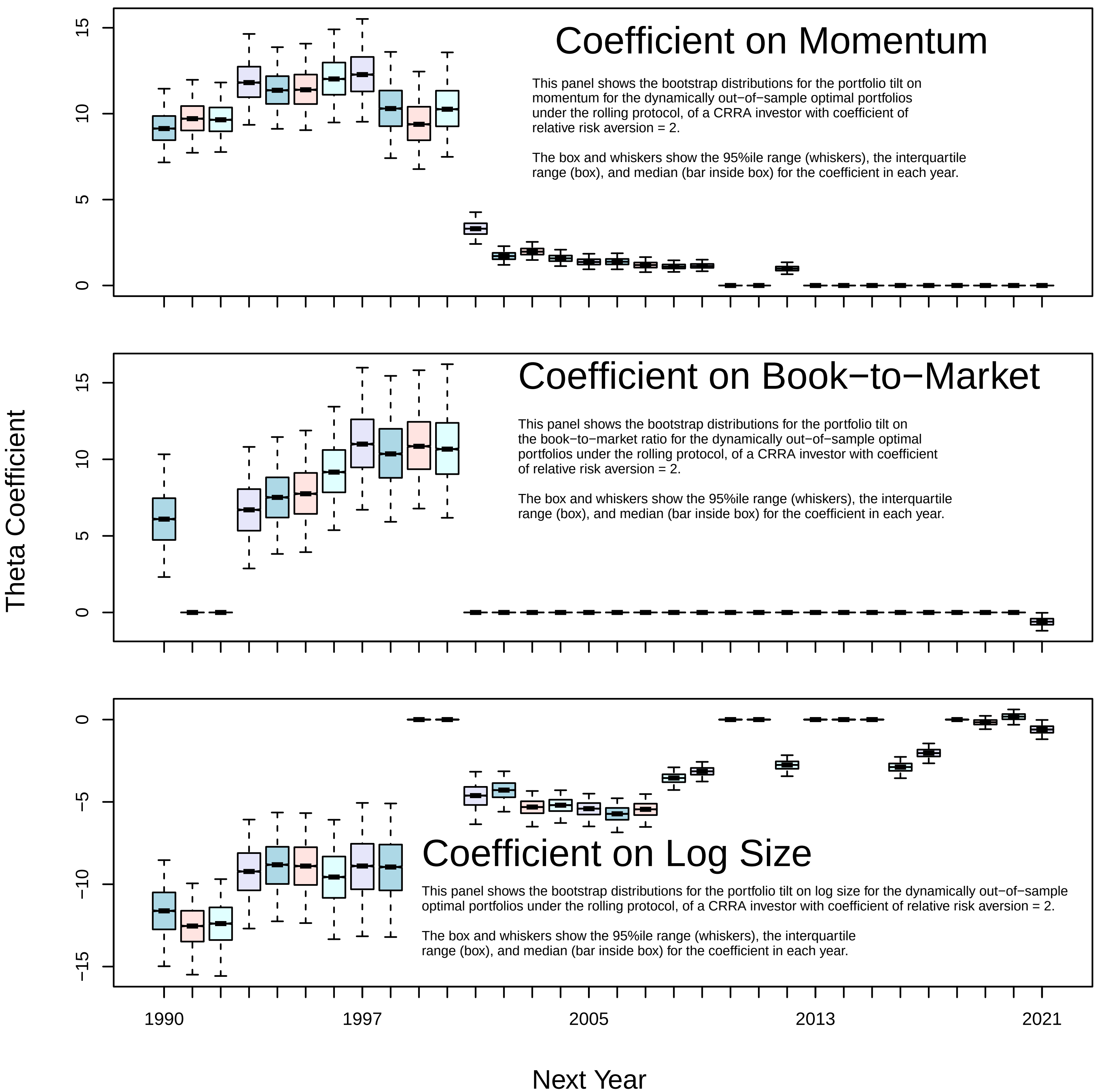

Figure 1. Characteristic tilts (θ) over time. Sampling distributions of the θ coefficient on (standardized) characteristics from the optimal model over the preceding 180 out-of-sample months, used to construct the optimal portfolio in the indicated year.

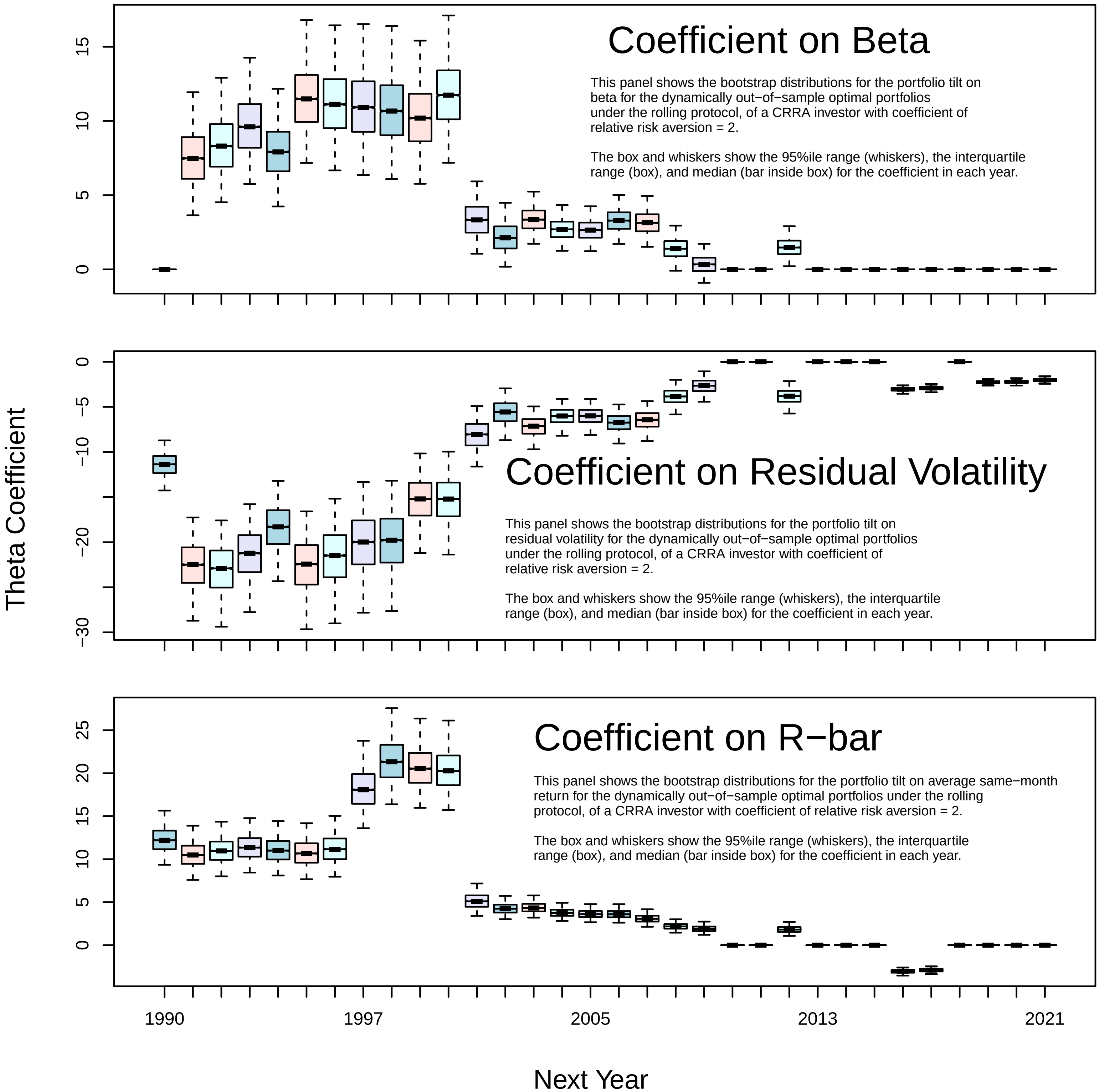

Figure 1 (Cont'd.). Characteristic tilts (θ) over time. Sampling distributions of the θ coefficient on (standardized) characteristics from the optimal model over the preceding 180 out-of-sample months, used to construct the optimal portfolio in the indicated year.

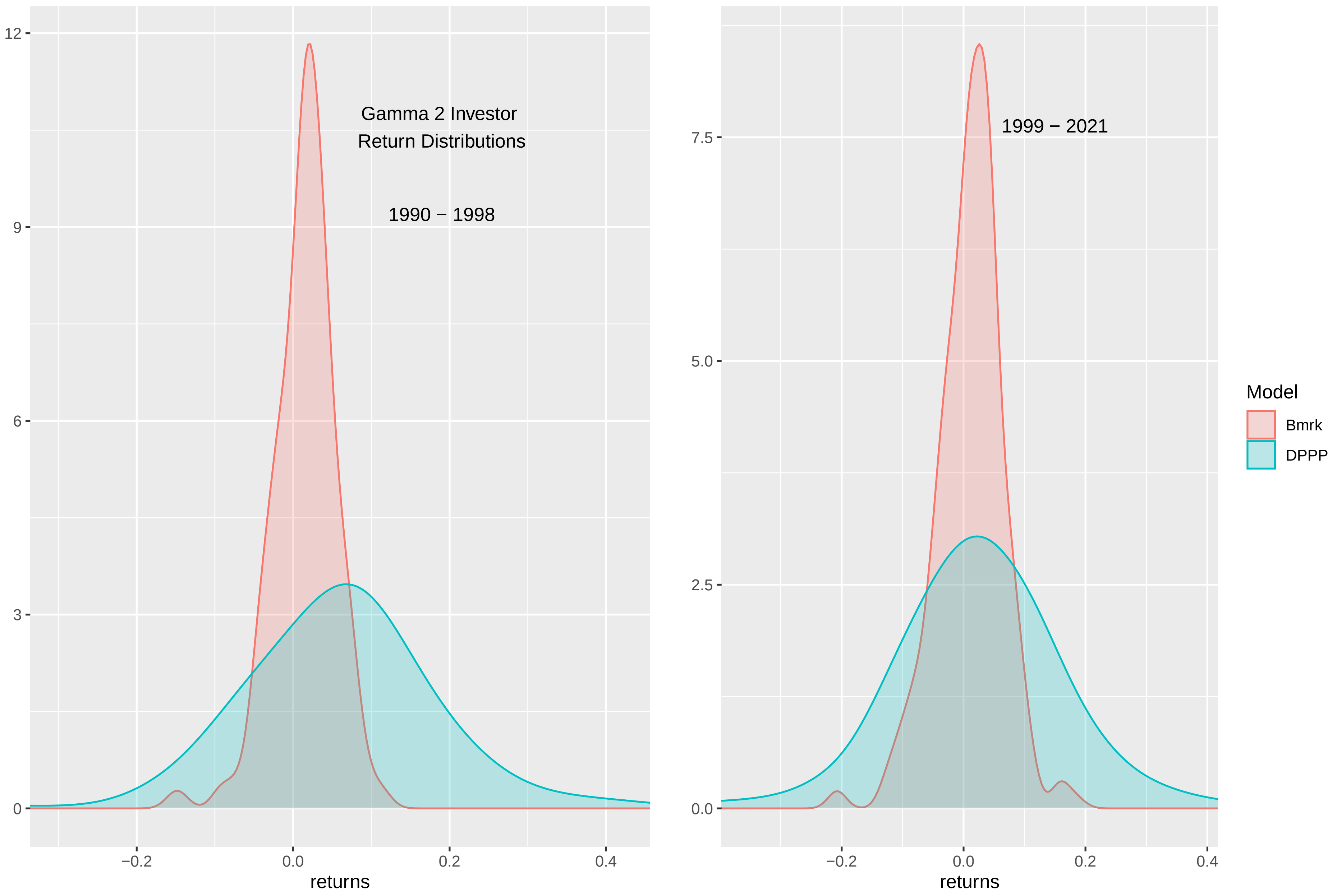

Figure 2 Portfolio return densities in the 2 subperiods. "DPPP" is the optimal dynamic parametric portfolio under the updating protocol–selected at the beginning of each year. "Bmrk" is the preferred benchmark in the subperiod. For this power utility investor with coefficient of relative risk aversion, γ, $=$ 2 that is the value-weighted portfolio of all stocks in the first subperiod and the equally-weighted portfolio of all stocks in the second subperiod.

Table 1

Optimal γ^* and portfolio (characteristic sets):
Sampling properties of certainty equivalent returns
For investor with power utility and coefficient of relative risk aversion, $\gamma = 2$.

Weight tilts (θ) are estimated for 63 characteristic sets under each of 14 values of the loss function curvature (γ^*), using both rolling and updating protocols. Of these 882 cases that with the highest 1%ile value of the out of sample certainty equivalent is reported for each of three investors in basis points per month. The characteristic symbols are: ζ: momentum, V: book-to-market ratio, S: log size, β: from lagged 60-month market model, $\bar{r}$: average same-month return over the previous 5 years, σ_ϵ: standard deviation of lagged 60-month market model residual.

	Updating Protocol					Rolling Protocol				
Next	Optimal		Certainty Equivalent			Optimal		Certainty Equivalent		
Year	Chars	γ^*	1%ile	Mean	Std Dev	Chars	γ^*	1%ile	Mean	Std Dev
1990	**VWI**		113.3	120.7	3.2	**VWI**		113.3	120.7	3.2
1990	**EWI**		145.0	149.1	1.8	**EWI**		145.0	149.1	1.8
1990	ζ,V,S,$\bar{r}$,σ_ϵ	2	-10,000.0	-4,146.6	5,245.0	ζ,V,S,$\bar{r}$,σ_ϵ	2	-10,000.0	-437.6	3,148.4
1990	ζ,V,S,$\bar{r}$,σ_ϵ	3	527.8	625.5	46.1	ζ,V,S,$\bar{r}$,σ_ϵ	3	523.1	622.2	46.0
1991	ζ,S,$\bar{r}$,σ_ϵ	3	497.7	589.7	43.5	ζ,S,β,$\bar{r}$,σ_ϵ	3	498.4	597.8	46.1
1992	ζ,S,$\bar{r}$,σ_ϵ	3	512.1	603.4	43.4	ζ,S,β,$\bar{r}$,σ_ϵ	3	497.4	601.6	48.1
1993	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	512.3	615.5	48.1	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	486.8	597.4	51.3
1994	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	518.2	619.6	47.3	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	493.3	606.4	52.3
1995	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	500.8	601.9	46.2	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	471.1	593.3	54.2
1996	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	489.7	584.7	44.2	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	496.1	615.5	54.3
1997	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	477.8	568.1	42.6	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	503.1	620.6	54.5
1998	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	503.9	598.6	44.3	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	540.8	670.4	60.4
1999	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	477.0	565.3	42.2	ζ,V,β,$\bar{r}$,σ_ϵ	3	488.7	624.1	61.9
2000	ζ,V,S,β,$\bar{r}$,σ_ϵ	3	472.2	562.5	42.9	ζ,V,β,$\bar{r}$,σ_ϵ	3	485.7	643.8	240.0
2001	ζ,S,β,$\bar{r}$,σ_ϵ	3	443.8	530.1	45.3	ζ,S,β,$\bar{r}$,σ_ϵ	7	229.4	322.9	39.9
2002	ζ,S,β,$\bar{r}$,σ_ϵ	3	415.5	503.0	44.4	ζ,S,β,$\bar{r}$,σ_ϵ	8	199.7	278.5	35.1
2003	ζ,S,β,$\bar{r}$,σ_ϵ	3	432.4	521.3	44.9	ζ,S,β,$\bar{r}$,σ_ϵ	8	219.1	303.6	38.1
2004	ζ,S,β,$\bar{r}$,σ_ϵ	3	401.5	483.8	41.7	ζ,S,β,$\bar{r}$,σ_ϵ	8	200.9	283.1	36.6
2005	ζ,S,β,$\bar{r}$,σ_ϵ	3	395.8	476.4	40.6	ζ,S,β,$\bar{r}$,σ_ϵ	8	181.7	265.6	36.8
2006	ζ,S,β,$\bar{r}$,σ_ϵ	3	388.9	468.3	39.5	ζ,S,β,$\bar{r}$,σ_ϵ	8	177.9	260.1	36.3
2007	ζ,S,β,$\bar{r}$,σ_ϵ	3	374.5	450.9	37.9	ζ,S,β,$\bar{r}$,σ_ϵ	8	154.8	235.2	35.0
2008	ζ,S,β,$\bar{r}$,σ_ϵ	3	345.9	415.7	35.4	ζ,S,β,$\bar{r}$,σ_ϵ	9	115.0	181.5	28.2
2009	ζ,S,β,$\bar{r}$,σ_ϵ	3	339.8	410.5	35.1	ζ,S,β,$\bar{r}$,σ_ϵ	10	87.7	146.7	26.0
2010	ζ,S,β,$\bar{r}$,σ_ϵ	3	305.7	376.1	35.1	**EWI**		72.8	77.6	2.1
2011	ζ,S,β,$\bar{r}$,σ_ϵ	3	306.6	376.0	34.4	**EWI**		69.2	74.1	2.1
2012	ζ,S,β,$\bar{r}$,σ_ϵ	3	306.2	375.4	34.1	ζ,S,β,$\bar{r}$,σ_ϵ	9	58.6	128.2	30.0
2013	ζ,S,β,$\bar{r}$,σ_ϵ	3	299.6	366.8	33.0	**EWI**		50.3	55.4	2.1
2014	ζ,S,β,$\bar{r}$,σ_ϵ	3	301.4	367.4	32.4	**EWI**		69.7	74.6	2.1
2015	ζ,S,β,$\bar{r}$,σ_ϵ	3	284.5	348.2	31.0	**EWI**		65.9	70.5	2.0
2016	ζ,S,β,$\bar{r}$,σ_ϵ	3	279.0	339.6	29.9	S,$\bar{r}$,σ_ϵ	6	84.8	114.0	13.8
2017	ζ,S,β,$\bar{r}$,σ_ϵ	3	273.7	333.0	29.1	S,$\bar{r}$,σ_ϵ	7	77.2	96.2	9.1
2018	ζ,S,β,$\bar{r}$,σ_ϵ	3	260.7	317.3	27.8	**EWI**		81.0	85.1	1.8
2019	ζ,S,β,$\bar{r}$,σ_ϵ	4	249.3	295.1	20.9	S,σ_ϵ	16	63.0	73.3	4.5
2020	ζ,S,β,$\bar{r}$,σ_ϵ	4	243.1	287.3	20.3	S,σ_ϵ	22	68.7	80.0	4.9
2021	ζ,S,β,$\bar{r}$,σ_ϵ	4	236.3	278.6	19.3	V,S,σ_ϵ	22	69.5	84.5	6.5

Table 2

Sampling properties of out-of-sample Portfolio Performance Statistics
108-month out-of-sample period, 1990 – 1998

Sampling properties of dynamic optimal PPPs. Portfolio characteristic tilts from the best out-of-sampling performer over the relevant preceding period (shown in Table 1) each year. $\mathcal{CE}_2$ is the certainty equivalent return in basis points per month for a power utility investor with coefficient of relative risk aversion $(\gamma) = 2$. $E(r)$, σ, Median, IQR, and MIN are the mean monthly return, the standard deviation of monthly returns, the median monthly return, the interquartile range of monthly returns, and the minimum monthly return–all expressed in basis points per month.
SKEW and KURT are the return skewness and kurtosis measures, and SR is the Sharpe ratio.
Results are for the first 9-year out-of-sample subperiod (1990 – 1998).

Panel A: Benchmark portfolios

	VWI							EWI						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_2$	128.8	4.0	121.0	126.1	128.8	131.5	136.6	109.4	2.3	104.8	107.9	109.4	110.9	113.9
$E(r)$	144.2	4.0	136.2	141.5	144.2	146.9	152.0	128.6	2.3	124.1	127.1	128.6	130.1	133.1
σ	388.6	4.6	379.6	385.5	388.6	391.7	397.7	430.0	2.3	425.1	428.3	430.0	431.6	434.9
Median	167.4	11.6	144.9	159.3	167.6	175.3	189.7	187.5	9.8	168.1	180.0	187.6	194.1	206.5
IQR	469.9	22.4	426.6	454.5	469.8	484.9	514.7	508.7	17.5	474.6	496.7	508.7	520.5	543.3
MIN	-1,482.9	59.1	-1,602.5	-1,521.9	-1,481.9	-1,442.2	-1,368.6	-1,747.3	28.2	-1,803.2	-1,766.3	-1,747.3	-1,728.2	-1,691.9
SKEW	-6.0	2.9	-11.6	-8.0	-6.0	-4.0	-0.4	-13.7	2.2	-18.0	-15.2	-13.7	-12.2	-9.3
KURT	26.4	4.6	17.4	23.3	26.4	29.4	35.6	35.2	2.5	30.3	33.5	35.2	36.9	40.2
SR	0.9337	0.0364	0.8627	0.9095	0.9337	0.9581	1.0051	0.7158	0.0187	0.6787	0.7034	0.7159	0.7284	0.7520

Panel B: Dynamic PPP

	Dyn. Opt. Updating							Dyn. Opt. Rolling						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_2$	427.7	50.9	328.6	393.6	428.1	461.1	529.0	497.2	246.8	291.1	444.1	507.7	570.1	688.4
$E(r)$	587.9	54.4	486.4	550.1	586.1	623.1	700.9	900.1	93.5	725.2	835.7	897.0	960.5	1,094.8
σ	1,254.8	88.6	1,090.0	1,193.7	1,251.6	1,311.1	1,441.2	1,952.8	129.8	1,714.8	1,862.7	1,947.4	2,035.4	2,221.6
Median	593.3	85.2	431.2	535.3	591.7	649.8	767.1	878.5	136.0	617.7	786.6	877.4	966.9	1,153.3
IQR	1,495.0	157.8	1,200.8	1,385.5	1,490.0	1,598.4	1,822.1	2,463.4	247.0	1,995.5	2,294.0	2,456.5	2,627.1	2,961.3
MIN	-3,887.9	705.9	-5,336.6	-4,344.9	-3,871.2	-3,399.8	-2,567.2	-5,145.8	1,160.3	-7,575.9	-5,901.6	-5,063.5	-4,286.7	-3,177.4
SKEW	-0.4	5.3	-10.9	-4.1	-0.4	3.2	10.2	1.1	5.7	-10.0	-2.7	1.0	4.9	12.5
KURT	24.1	13.4	-1.2	14.9	23.9	32.9	50.7	11.8	14.2	-15.2	1.9	11.4	21.2	40.8
SR	1.5281	0.1383	1.2626	1.4353	1.5266	1.6191	1.8050	1.5400	0.1391	1.2741	1.4451	1.5369	1.6333	1.8157

Table 3

Out-of-Sample 6-factor Fama-French regressions

$$r_{i,t} - r_f = \alpha + \beta_1 \cdot (R_{m,t} - r_f) + \beta_2 \cdot \text{HML} + \ \beta_3 \cdot \text{SMB} + \beta_4 \cdot \text{MOM} + \beta_5 \cdot \text{RMW} + \beta_6 \cdot \text{CMA} + \epsilon_{i,t}$$

For power utility investor with coefficient of relative risk aversion, $\gamma = 2$. Monthly returns; α in basis points per month.

Panel A. Subperiod 1: 1990 - 1998

	Updating protocol							Rolling protocol						
Coefficient	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
α / Orthog.	263.75	54.97	156.53	226.66	263.04	299.71	371.25	387.20	94.63	209.65	323.21	385.11	450.49	577.66
Mkt	-0.36	0.17	-0.69	-0.47	-0.36	-0.25	-0.04	-0.51	0.27	-1.05	-0.69	-0.51	-0.33	-0.02
HML	3.16	0.43	2.35	2.86	3.15	3.44	4.02	5.83	0.66	4.59	5.39	5.82	6.27	7.14
SMB	1.68	0.28	1.12	1.48	1.68	1.87	2.23	0.78	0.43	-0.06	0.50	0.78	1.06	1.64
MOM	2.62	0.25	2.14	2.44	2.61	2.78	3.13	3.80	0.39	3.04	3.53	3.80	4.06	4.58
RMW	0.93	0.41	0.14	0.65	0.92	1.20	1.75	1.42	0.74	-0.03	0.93	1.41	1.92	2.89
CMA	-1.67	0.53	-2.73	-2.01	-1.66	-1.30	-0.66	-4.08	0.86	-5.81	-4.65	-4.07	-3.50	-2.42

Panel B. Subperiod 1: 1990 - 1998 Updating Protocol: Decompositions

	% of Portfolio Mean due to:							% of Portfolio Variance due to:						
Coefficient	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
α / Orthog.	47.86	7.07	32.94	43.41	48.22	52.74	60.76	53.16	4.26	45.09	50.24	53.12	56.00	61.72
Mkt	-6.64	3.15	-13.04	-8.71	-6.56	-4.50	-0.65	1.52	1.17	0.03	0.63	1.27	2.17	4.42
HML	14.27	2.02	10.63	12.90	14.18	15.54	18.52	37.75	7.77	23.31	32.39	37.46	42.86	53.65
SMB	-7.90	1.62	-11.23	-8.94	-7.85	-6.79	-4.90	13.45	4.34	5.74	10.36	13.16	16.25	22.65
MOM	48.11	5.57	38.05	44.32	47.83	51.57	59.98	34.07	4.65	25.09	30.97	34.05	37.17	43.41
RMW	7.35	3.29	1.10	5.12	7.26	9.49	14.02	1.22	0.92	0.04	0.52	1.03	1.72	3.51
CMA	-3.06	0.97	-5.01	-3.68	-3.05	-2.40	-1.23	6.61	3.73	1.03	3.84	6.11	8.75	15.32

Panel C. Subperiod 2: 1999 - 2021

	Updating protocol							Rolling protocol						
Coefficient	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
α / Orthog.	80.86	34.66	14.47	56.94	80.18	104.21	150.37	99.24	30.89	40.38	78.48	98.04	119.61	161.97
Mkt	0.91	0.11	0.70	0.84	0.91	0.98	1.12	1.27	0.09	1.10	1.20	1.27	1.33	1.47
HML	1.38	0.16	1.07	1.27	1.38	1.49	1.70	0.74	0.14	0.47	0.65	0.73	0.83	1.02
SMB	0.62	0.22	0.18	0.47	0.62	0.77	1.04	-1.14	0.26	-1.69	-1.31	-1.13	-0.95	-0.67
MOM	1.01	0.15	0.71	0.91	1.01	1.12	1.32	0.50	0.14	0.23	0.41	0.50	0.59	0.78
RMW	1.26	0.22	0.83	1.11	1.26	1.41	1.70	0.01	0.31	-0.59	-0.19	0.02	0.22	0.61
CMA	-0.22	0.25	-0.70	-0.39	-0.22	-0.05	0.20	-0.13	0.25	-0.62	-0.29	-0.12	0.04	0.35

Table 4

Sampling properties of out-of-sample Portfolio Performance Statistics
276-month out-of-sample period, 1999 – 2021

Sampling properties of dynamic optimal PPPs. Portfolio characteristic tilts from the best out-of-sampling performer over the relevant preceding period (shown in Table 1) each year. $\mathcal{CE}_2$ is the certainty equivalent return in basis points per month for a power utility investor with coefficient of relative risk aversion $(\gamma) = 2$. $E(r)$, σ, Median, IQR, and MIN are the mean monthly return, the standard deviation of monthly returns, the median monthly return, the interquartile range of monthly returns, and the minimum monthly return–all expressed in basis points per month.
SKEW and KURT are the return skewness and kurtosis measures, and SR is the Sharpe ratio.
Results are for the second 23-year out-of-sample subperiod (1999 – 2021).

Panel A: Benchmark portfolios

	VWI							EWI						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_2$	60.0	3.5	53.1	57.6	60.0	62.4	66.9	76.6	1.7	73.4	75.4	76.6	77.7	79.8
$E(r)$	79.2	3.5	72.4	76.8	79.2	81.6	85.9	106.8	1.7	103.6	105.6	106.8	107.9	110.0
σ	433.3	4.2	425.2	430.4	433.3	436.3	441.6	542.7	2.1	538.6	541.3	542.7	544.1	546.8
Median	124.0	8.0	107.9	118.6	124.1	129.4	139.4	141.9	6.7	128.8	137.4	141.9	146.4	155.3
IQR	511.2	13.9	484.6	501.8	510.9	520.6	538.8	655.0	12.1	612.0	626.8	635.0	642.9	659.3
MIN	-1,667.4	78.6	-1,819.4	-1,720.6	-1,667.6	-1,613.6	-1,514.3	-2119.7	42.4	-2,206.5	-2,147.9	-2,118.6	-2,090.3	-2,039.7
SKEW	-10.3	1.8	-13.9	-11.5	-10.4	-9.1	-6.8	-6.5	1.2	-8.9	-7.3	-6.5	-5.6	-4.1
KURT	27.9	3.8	20.6	25.3	27.9	30.4	35.4	32.5	1.5	29.5	31.5	32.5	33.5	35.5
SR	0.5245	0.0286	0.4687	0.5052	0.5246	0.5436	0.5807	0.5959	0.0105	0.5756	0.5887	0.5957	0.6029	0.6165

Panel B: Dynamic PPP

	Dyn. Opt. Updating							Dyn. Opt. Rolling						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_2$	-1.5	172.3	-110.2	-24.9	5.9	33.5	82.9	-5,017.2	4,7619.8	-10,000	-10,000	-2,099.6	-222.1	4.1
$E(r)$	242.7	32.8	180.1	220.1	242.2	264.3	309.4	175.2	28.1	121.4	156.3	174.7	193.0	232.9
σ	1,472.3	72.6	1,339.3	1,421.4	1,469.2	1,519.9	1,622.7	1,370.2	144.2	1,120.7	1,269.2	1,356.7	1,459.4	1,685.5
Median	233.0	52.2	133.8	196.7	232.7	267.9	337.9	149.2	22.8	107.0	132.9	148.5	164.9	194.5
IQR	1,718.9	113.3	1,510.8	1,639.5	1,714.3	1,792.7	1,954.6	748.9	37.6	680.4	722.2	747.6	773.7	826.2
MIN	-5,490.7	933.4	-7,756.8	-5,958.2	-5,352.3	-4,856.6	-4,072.5	-10,082.7	1,862.8	-14,309.7	-11,127.4	-9,861.2	-8,787.2	-7,071.3
SKEW	0.6	3.2	-5.7	-1.5	0.7	2.8	6.8	1.9	2.1	-2.3	0.5	1.9	3.3	6.0
KURT	40.2	11.5	18.6	32.3	39.7	47.7	63.2	209.8	23.5	164.1	193.6	209.6	225.4	256.9
SR	0.5415	0.0750	0.3971	0.4895	0.5414	0.5931	0.6902	0.4130	0.0726	0.2717	0.3641	0.4127	0.4617	0.5566

Internet Appendix *for*
An Empirical Assessment of Characteristics and Optimal Portfolios

This appendix provides supporting results for the paper An Empirical Assessment of Characteristics and Optimal Portfolios.
Contents:

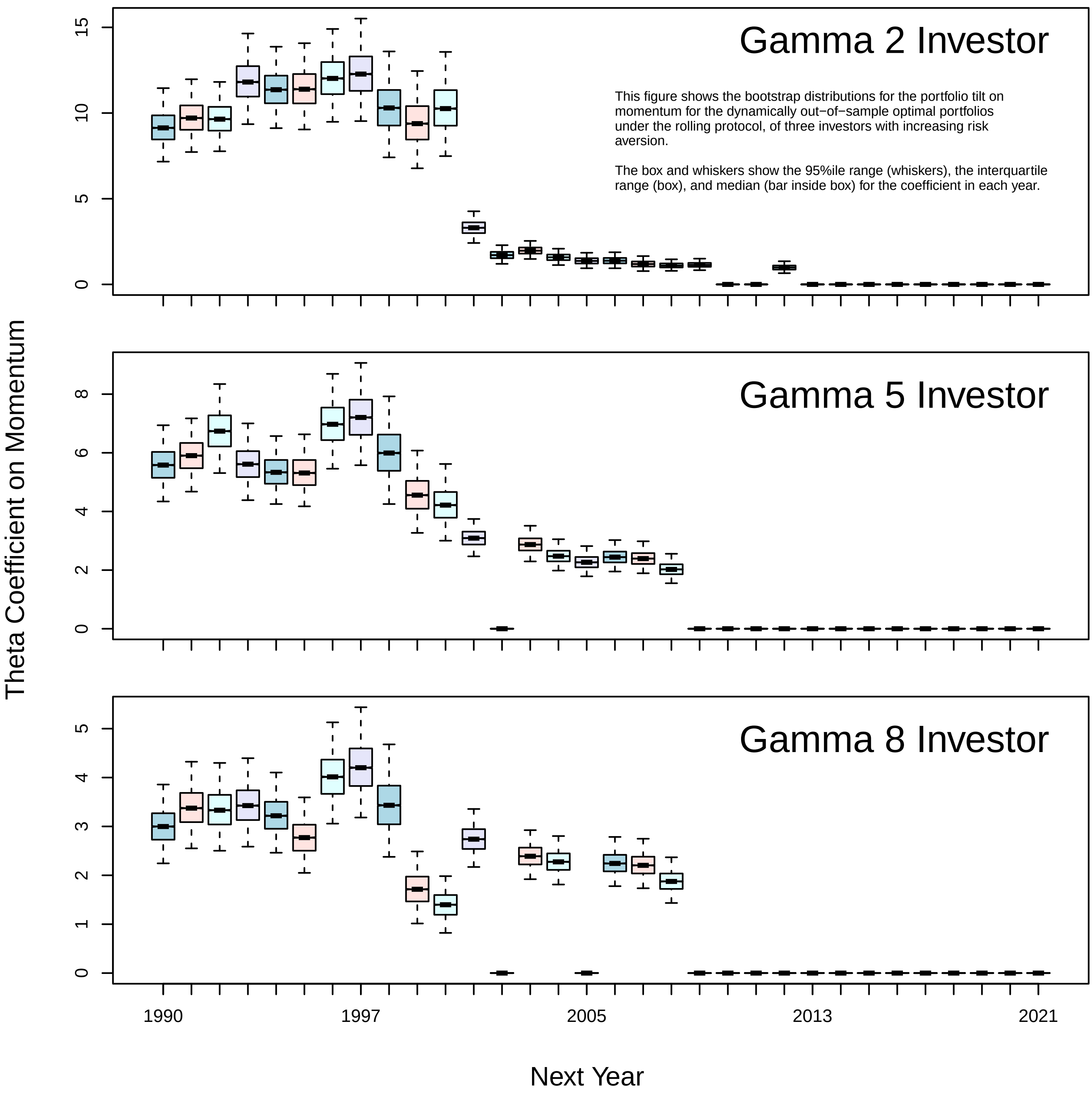

Figure IA-1. Momentum tilt. Sampling distributions of the θ coefficient on (standardized) momentum from the optimal model over the preceding 180 months–out-of-sample, used to construct the optimal portfolio in the indicated year.

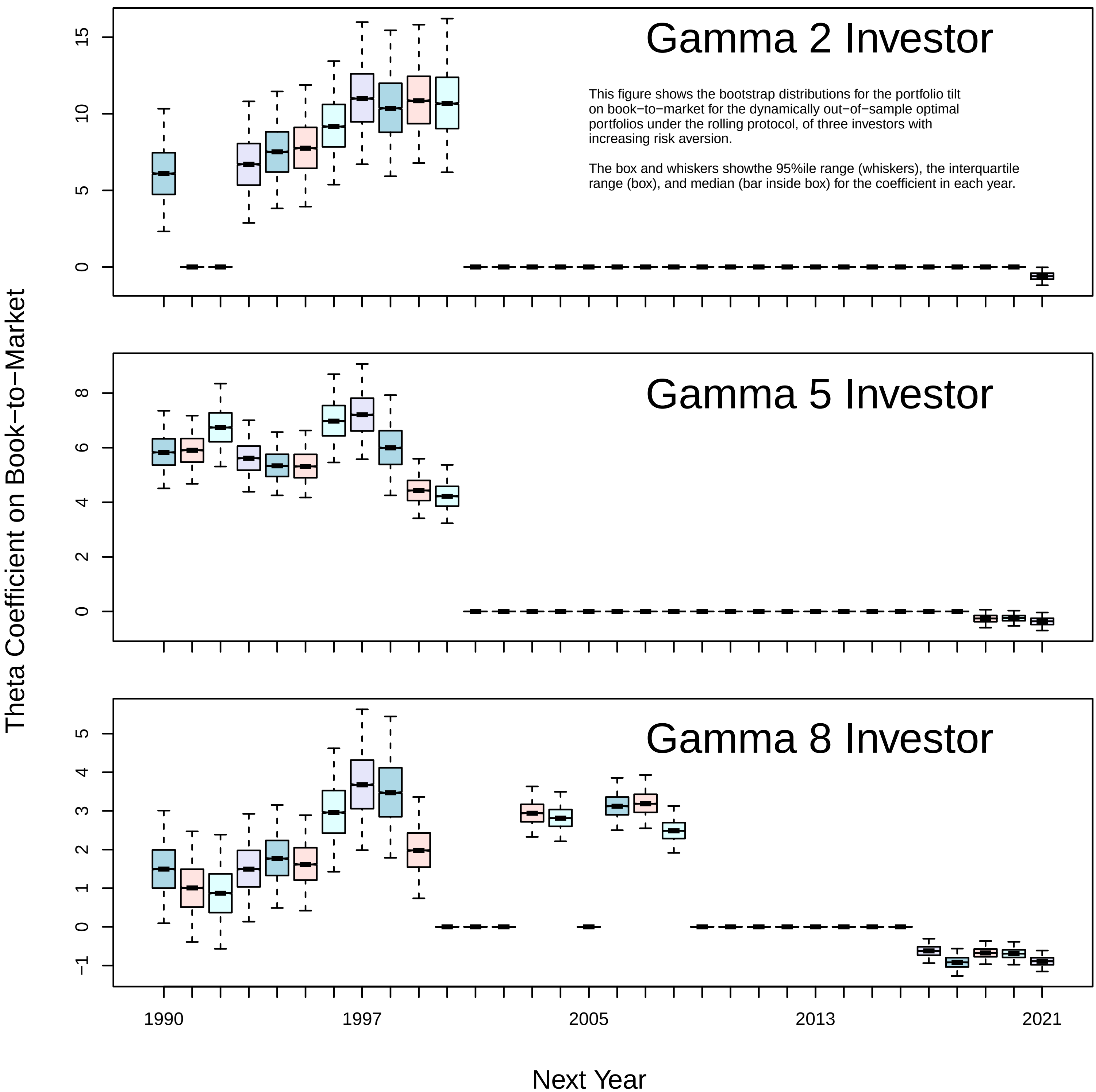

Figure IA-2. Value tilt. Sampling distributions of the θ coefficient on the (standardized) book-to-market ratio from the optimal model over the preceding 180 months–out-of-sample, used to construct the optimal portfolio in the indicated year.

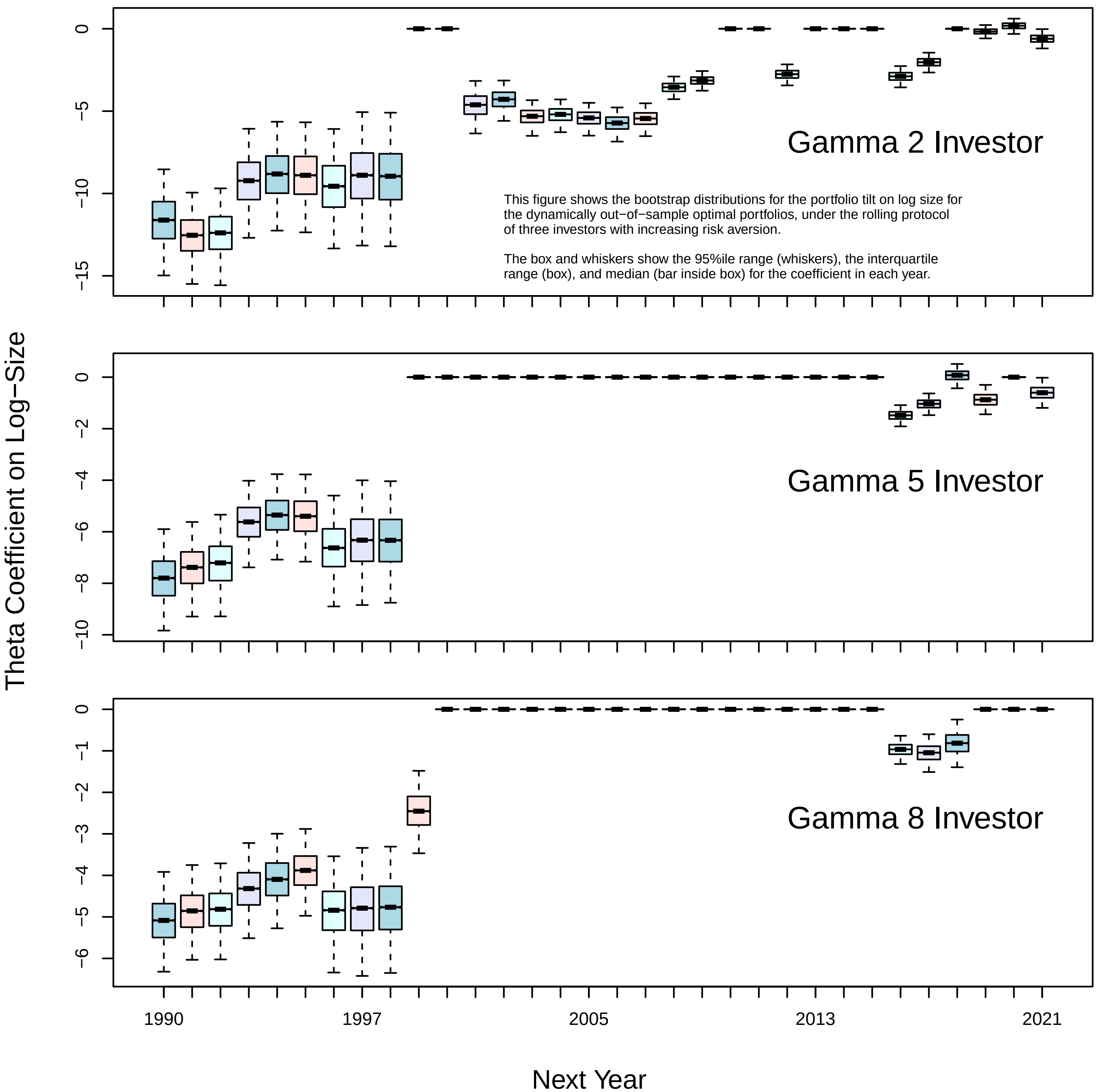

Figure IA-3. Size tilt. Sampling distributions of the θ coefficient on (standardized) log market capitalization from the optimal model over the preceding 180 months–out-of-sample, used to construct the optimal portfolio in the indicated year.

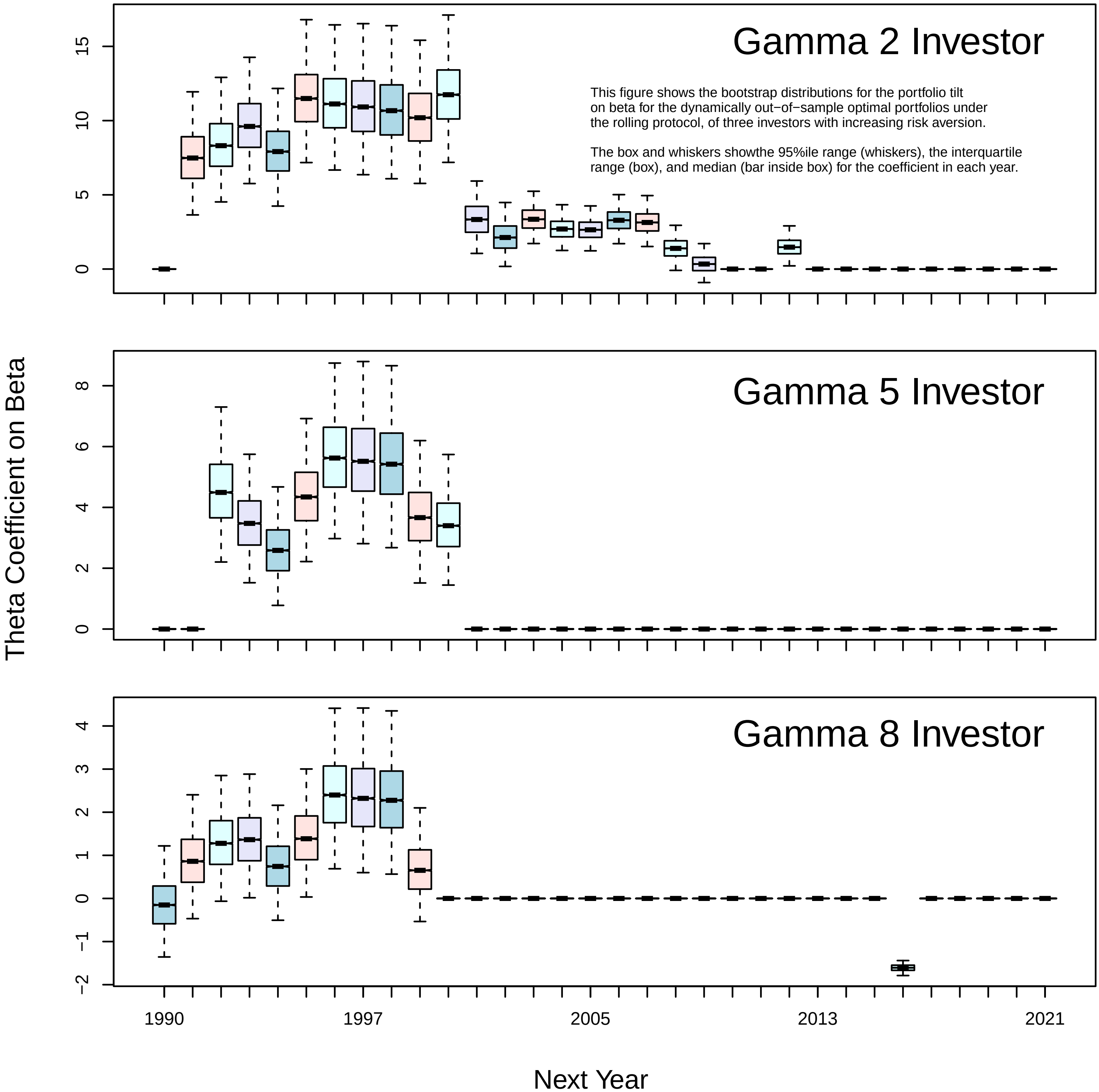

Figure IA-4. Beta tilt. Sampling distributions of the θ coefficient on (standardized) beta from the optimal model over the preceding 180 months–out-of-sample, used to construct the optimal portfolio in the indicated year.

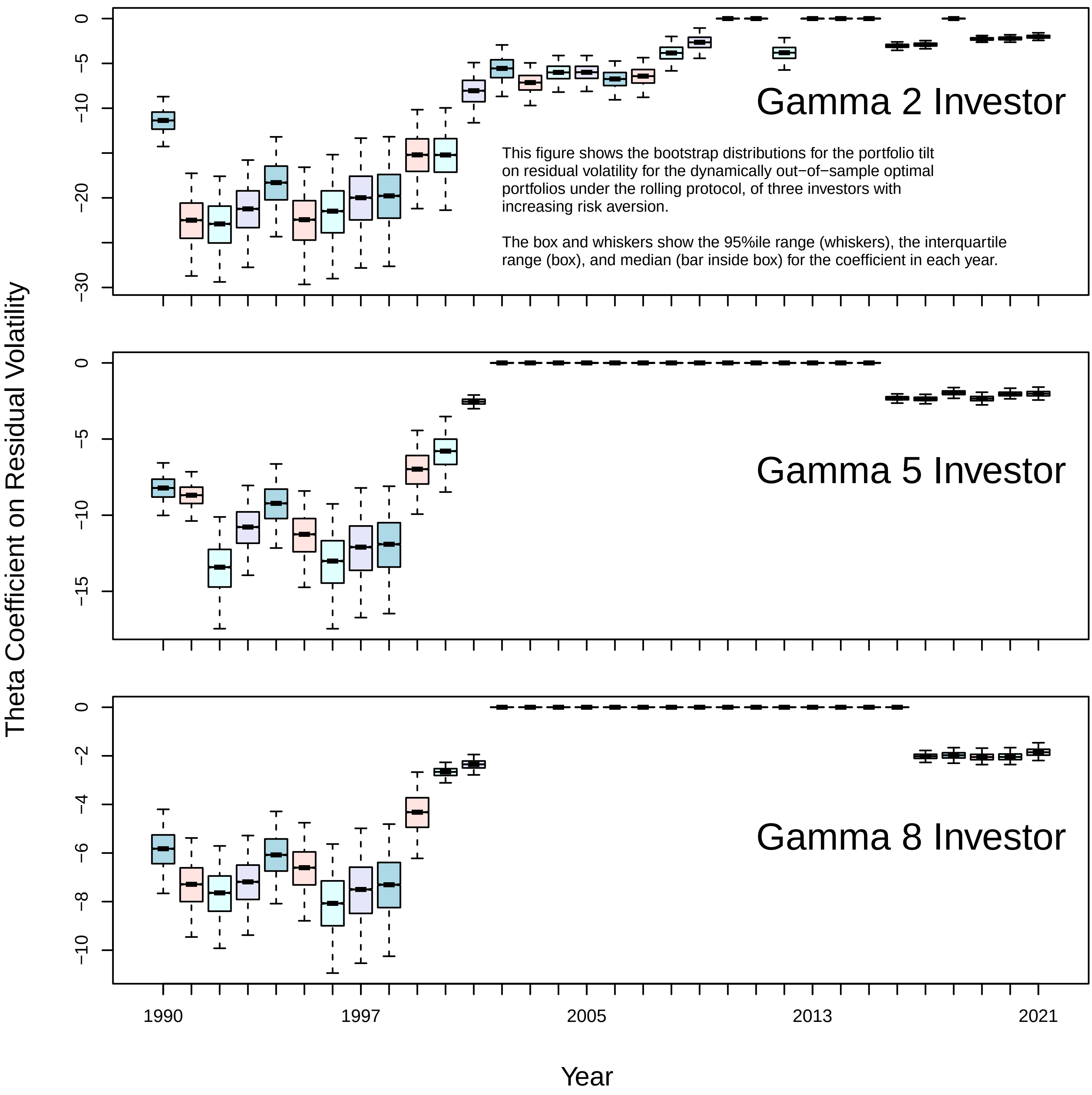

Figure IA-5. Residual volatility tilt. Sampling distributions of the θ coefficient on (standardized) residual standardized deviation from the optimal model over the preceding 180 months–out-of-sample, used to construct the optimal portfolio in the indicated year.

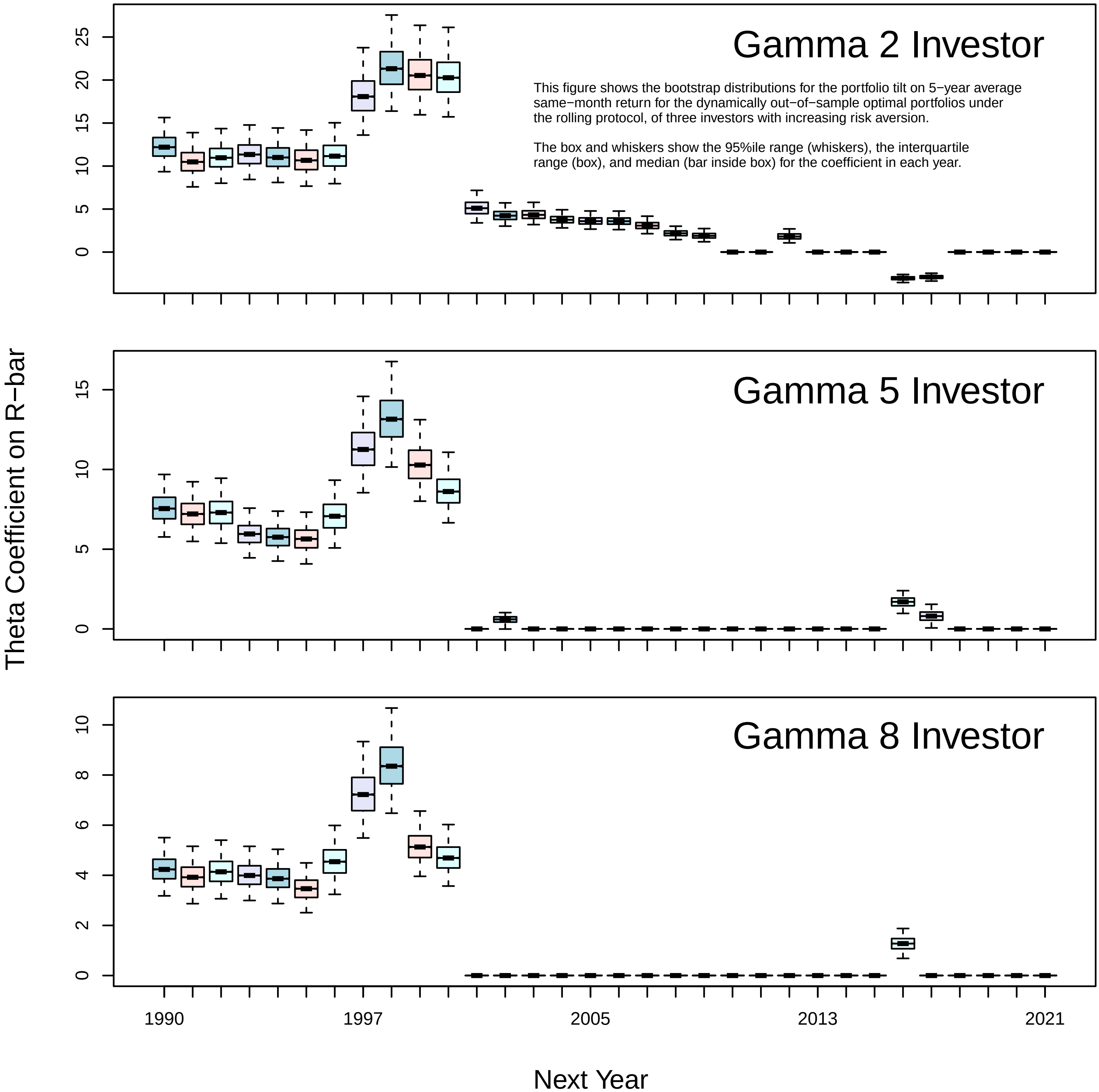

Figure IA-6. Average same-month return tilt. Sampling distributions of the θ coefficient on the (standardized) average same-month return from the optimal model over the preceding 180 months–out-of-sample, used to construct the optimal portfolio in the indicated year.

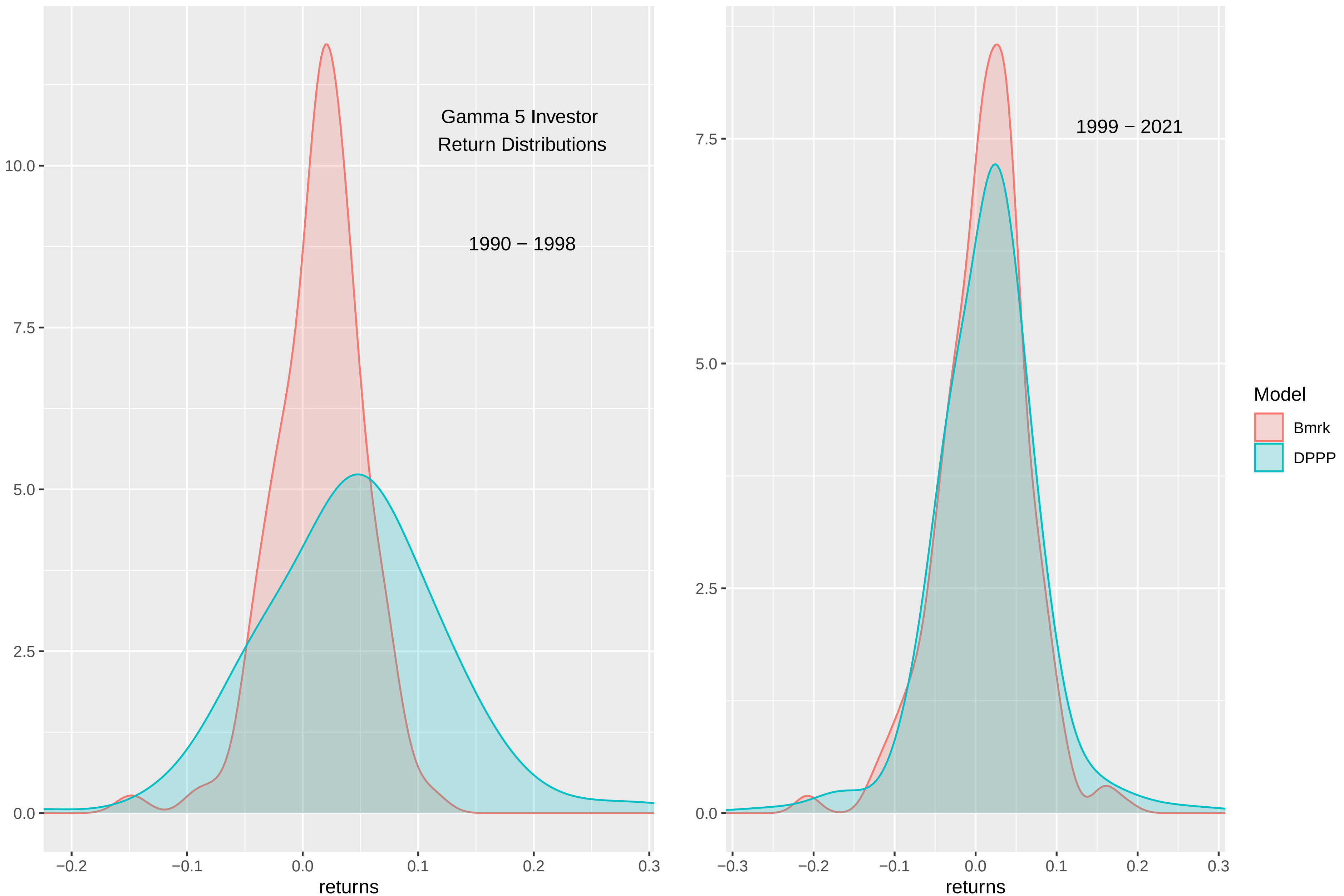

Figure IA-7. Portfolio return densities in the 2 subperiods. "DPPP" is the optimal dynamic parametric portfolio under the updating protocol–selected at the beginning of each year. "Bmrk" is the preferred benchmark in the subperiod. For this power utility investor with coefficient of relative risk aversion, $\gamma = 5$: the value-weighted portfolio of all stocks in the first subperiod and the equally-weighted portfolio of all stocks in the second subperiod.

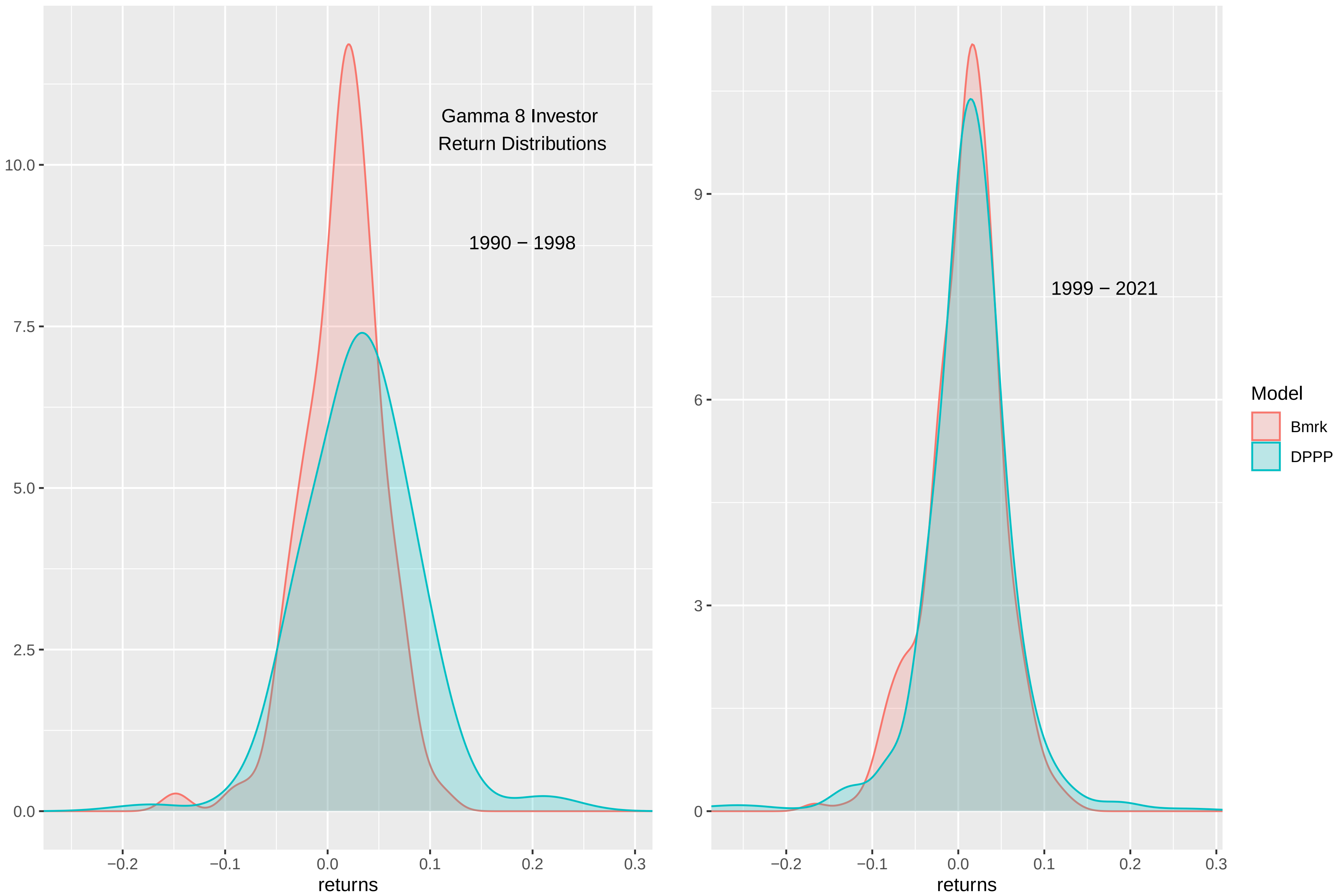

Figure IA-8. Portfolio return densities in the 2 subperiods. "DPPP" is the optimal dynamic parametric portfolio under the updating protocol–selected at the beginning of each year. "Bmrk" is the preferred benchmark in the subperiod. For this power utility investor with coefficient of relative risk aversion, $\gamma = 8$: the value-weighted portfolio of all stocks in both subperiods.

Table IA-1
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months [−18, −6] to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
December-1959	469	456	411	26,277	198,482
January-1960	472	457	412	26,862	181,843
February-1960	473	456	411	26,738	186,863
March-1960	473	454	409	26,623	179,322
April-1960	473	455	410	25,752	177,732
May-1960	476	460	414	25,538	178,108
June-1960	476	460	414	26,988	180,770
July-1960	478	459	414	25,820	177,441
August-1960	478	462	416	25,698	178,343
September-1960	480	462	416	24,024	168,739
October-1960	481	459	414	23,952	168,416
November-1960	484	463	417	23,250	175,208
December-1960	487	464	418	25,082	187,619
January-1961	491	471	424	25,798	195,168
February-1961	495	474	427	26,802	209,228
March-1961	497	481	433	27,547	207,533
April-1961	499	481	433	28,017	203,797
May-1961	534	503	453	25,183	200,238
June-1961	535	503	453	24,541	194,714
July-1961	536	504	454	24,544	198,551
August-1961	536	502	452	25,523	201,934
September-1961	537	499	450	25,420	200,271
October-1961	538	499	450	24,567	208,202
November-1961	539	505	455	25,394	203,906
December-1961	542	509	459	25,628	209,549
January-1962	542	510	459	25,846	202,489
February-1962	542	513	462	24,956	198,553
March-1962	542	510	459	25,344	204,148
April-1962	542	504	454	24,854	194,457
May-1962	547	501	451	23,209	179,199
June-1962	544	496	447	22,813	166,498
July-1962	546	504	454	23,391	168,964
August-1962	550	506	456	24,750	174,250
September-1962	550	500	450	25,253	166,191
October-1962	552	497	448	25,424	168,204
November-1962	558	513	462	25,970	179,241
December-1962	562	515	464	24,917	179,326

Table IA-1
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18, -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1963	565	525	473	24,011	180,719
February-1963	571	531	478	25,234	174,471
March-1963	575	535	482	25,051	186,411
April-1963	575	536	483	25,278	191,703
May-1963	615	578	521	27,393	185,043
June-1963	617	578	521	26,202	184,613
July-1963	617	576	519	26,838	186,339
August-1963	619	583	525	25,667	187,841
September-1963	620	579	522	27,246	190,534
October-1963	620	582	524	27,743	184,228
November-1963	620	580	522	28,196	186,675
December-1963	622	580	522	28,130	195,023
January-1964	624	584	526	28,123	192,954
February-1964	624	585	527	28,013	194,858
March-1964	627	593	534	26,877	193,158
April-1964	630	593	534	28,518	193,054
May-1964	640	600	540	28,928	196,121
June-1964	643	604	544	28,013	195,869
July-1964	645	608	548	28,187	195,636
August-1964	643	604	544	29,715	192,946
September-1964	645	608	548	29,417	204,829
October-1964	647	610	549	31,185	207,468
November-1964	652	616	555	29,464	208,542
December-1964	653	618	557	29,775	199,523
January-1965	653	623	561	29,633	208,249
February-1965	655	628	566	29,416	210,972
March-1965	658	633	570	29,584	209,008
April-1965	657	632	569	31,899	219,375
May-1965	689	663	597	31,862	206,585
June-1965	696	666	600	27,030	186,960
July-1965	700	672	605	27,958	185,763
August-1965	704	677	610	28,860	191,818
September-1965	708	684	616	28,760	197,295
October-1965	709	686	618	29,458	204,931
November-1965	709	685	617	30,475	207,690
December-1965	713	692	623	31,363	211,730

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18, -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1966	716	698	629	32,702	206,904
February-1966	716	700	630	33,722	205,932
March-1966	716	699	630	32,872	206,818
April-1966	720	704	634	32,780	210,266
May-1966	727	703	633	31,020	196,721
June-1966	731	707	637	29,799	197,799
July-1966	734	707	637	30,509	195,896
August-1966	736	703	633	28,909	182,819
September-1966	742	708	638	27,808	179,487
October-1966	749	710	639	28,899	176,234
November-1966	753	715	644	28,886	181,985
December-1966	753	716	645	29,000	187,575
January-1967	756	733	660	31,000	197,309
February-1967	759	736	663	31,125	201,510
March-1967	763	743	669	32,895	210,937
April-1967	764	748	674	31,734	219,412
May-1967	767	748	674	33,070	216,716
June-1967	768	753	678	34,750	215,572
July-1967	1,145	978	881	22,155	152,183
August-1967	1,152	988	890	21,672	150,631
September-1967	1,164	1,006	906	22,514	148,847
October-1967	1,166	998	899	22,897	147,687
November-1967	1,172	995	896	23,500	147,199
December-1967	1,179	1,022	920	23,030	154,618
January-1968	1,180	1,030	927	23,385	148,004
February-1968	1,182	1,012	911	23,621	149,284
March-1968	1,190	1,017	916	23,925	145,821
April-1968	1,198	1,039	936	25,665	154,682
May-1968	1,210	1,079	972	23,599	152,028
June-1968	1,214	1,082	974	24,413	155,456
July-1968	1,213	1,074	967	24,750	152,260
August-1968	1,218	1,091	982	25,067	152,600
September-1968	1,222	1,109	999	25,398	158,815
October-1968	1,222	1,108	998	24,408	161,185
November-1968	1,229	1,118	1,007	25,642	167,999
December-1968	1,238	1,130	1,017	26,898	170,701

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months [−18, −6] to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1969	1,246	1,137	1,024	26,928	169,884
February-1969	1,262	1,131	1,018	26,786	161,004
March-1969	1,261	1,130	1,017	26,856	164,496
April-1969	1,271	1,130	1,017	27,114	163,191
May-1969	1,288	1,150	1,035	26,483	164,405
June-1969	1,299	1,126	1,014	26,082	152,692
July-1969	1,306	1,108	998	25,326	144,156
August-1969	1,304	1,109	999	25,400	150,713
September-1969	1,317	1,106	996	24,640	152,133
October-1969	1,322	1,129	1,017	25,568	156,599
November-1969	1,327	1,117	1,006	24,648	151,620
December-1969	1,333	1,107	997	23,962	155,108
January-1970	1,339	1,101	991	24,317	148,749
February-1970	1,347	1,112	1,001	24,340	147,026
March-1970	1,351	1,110	999	23,798	144,923
April-1970	1,360	1,063	957	22,715	131,445
May-1970	1,365	1,032	929	23,077	124,230
June-1970	1,372	1,018	917	21,954	120,225
July-1970	1,374	1,028	926	22,544	127,966
August-1970	1,378	1,046	942	22,515	130,708
September-1970	1,378	1,073	966	23,184	133,555
October-1970	1,385	1,057	952	22,788	132,833
November-1970	1,386	1,056	951	23,180	138,671
December-1970	1,399	1,076	969	24,929	148,933
January-1971	1,405	1,118	1,007	24,711	152,665
February-1971	1,403	1,134	1,021	24,393	153,459
March-1971	1,404	1,143	1,029	24,885	158,011
April-1971	1,405	1,143	1,029	26,004	165,055
May-1971	1,406	1,129	1,017	25,305	158,353
June-1971	1,412	1,124	1,012	25,476	161,626
July-1971	1,425	1,112	1,001	25,185	155,701
August-1971	1,432	1,127	1,015	25,146	164,238
September-1971	1,440	1,126	1,014	25,200	161,701
October-1971	1,444	1,111	1,000	24,645	155,283
November-1971	1,446	1,103	993	24,262	153,243
December-1971	1,445	1,129	1,017	24,960	168,139

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18,\ -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1972	1,451	1,156	1,041	25,397	170,630
February-1972	1,451	1,162	1,046	26,035	178,619
March-1972	1,453	1,166	1,050	25,239	178,408
April-1972	1,457	1,165	1,049	26,130	173,812
May-1972	1,465	1,161	1,045	27,016	174,563
June-1972	1,474	1,149	1,035	27,543	173,099
July-1972	1,476	1,151	1,036	27,312	172,140
August-1972	1,484	1,152	1,037	27,263	174,641
September-1972	1,494	1,145	1,031	26,729	178,604
October-1972	1,501	1,148	1,034	26,104	173,773
November-1972	1,510	1,156	1,041	28,031	182,792
December-1972	1,518	1,164	1,048	28,633	185,024
January-1973	1,528	1,159	1,044	27,869	168,607
February-1973	1,533	1,143	1,029	26,760	158,445
March-1973	1,543	1,142	1,028	26,338	157,846
April-1973	1,553	1,125	1,013	25,730	152,246
May-1973	1,554	1,101	991	25,436	147,990
June-1973	1,561	1,089	981	25,500	146,954
July-1973	1,567	1,125	1,013	25,530	154,840
August-1973	1,578	1,111	1,000	25,525	154,707
September-1973	1,582	1,134	1,021	27,683	166,756
October-1973	1,591	1,133	1,020	27,200	168,873
November-1973	1,600	1,071	964	26,118	151,040
December-1973	1,610	1,057	952	26,418	157,706
January-1974	1,621	1,107	997	27,054	153,915
February-1974	1,636	1,112	1,001	26,955	152,928
March-1974	1,648	1,113	1,002	27,224	153,504
April-1974	1,659	1,100	990	26,985	147,341
May-1974	1,673	1,078	971	26,537	140,207
June-1974	1,683	1,073	966	25,880	139,228
July-1974	1,692	1,053	948	27,050	138,117
August-1974	1,707	1,021	919	27,738	135,984
September-1974	1,714	993	894	27,176	129,658
October-1974	1,710	1,023	921	27,666	135,506
November-1974	1,716	1,002	902	27,793	138,147
December-1974	1,721	977	880	27,319	142,810

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months [−18, −6] to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1975	1,730	1,067	961	28,247	145,871
February-1975	1,742	1,074	967	29,013	154,957
March-1975	1,754	1,110	999	28,951	153,146
April-1975	1,757	1,119	1,008	28,627	157,762
May-1975	1,771	1,131	1,018	30,254	162,335
June-1975	1,771	1,161	1,045	29,722	163,768
July-1975	1,777	1,153	1,038	29,631	163,863
August-1975	1,779	1,117	1,006	29,780	169,567
September-1975	1,783	1,104	994	29,997	168,518
October-1975	1,790	1,108	998	30,760	176,060
November-1975	1,795	1,115	1,004	31,155	176,361
December-1975	1,753	1,106	996	31,388	177,837
January-1976	1,764	1,188	1,070	30,199	178,962
February-1976	1,749	1,217	1,096	30,195	180,812
March-1976	1,749	1,221	1,099	29,984	179,046
April-1976	1,751	1,215	1,094	31,017	177,453
May-1976	1,757	1,212	1,091	31,167	176,868
June-1976	1,762	1,214	1,093	31,839	187,299
July-1976	1,641	1,200	1,080	32,863	190,857
August-1976	1,645	1,194	1,075	33,569	187,032
September-1976	1,636	1,193	1,074	33,616	192,962
October-1976	1,640	1,182	1,064	34,004	190,791
November-1976	1,647	1,193	1,074	33,860	198,162
December-1976	1,641	1,221	1,099	33,942	205,271
January-1977	1,644	1,231	1,108	33,561	199,309
February-1977	1,642	1,222	1,100	33,363	195,582
March-1977	1,648	1,224	1,102	34,164	195,052
April-1977	1,656	1,227	1,105	34,688	197,363
May-1977	1,666	1,230	1,107	34,941	193,923
June-1977	1,668	1,254	1,129	35,488	197,351
July-1977	1,672	1,248	1,124	35,211	197,328
August-1977	1,677	1,234	1,111	36,013	197,855
September-1977	1,669	1,224	1,102	37,635	202,003
October-1977	1,672	1,224	1,102	36,772	197,232
November-1977	1,669	1,251	1,001	56,084	249,400
December-1977	2,829	1,775	1,420	44,712	171,134

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18, -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1978	2,837	1,748	1,399	45,627	167,855
February-1978	2,842	1,771	1,417	44,594	165,400
March-1978	2,842	1,819	1,456	45,117	169,908
April-1978	2,838	1,858	1,487	45,639	176,291
May-1978	2,832	1,878	1,503	45,201	178,126
June-1978	2,815	1,856	1,485	46,023	178,354
July-1978	2,803	1,880	1,504	46,795	185,598
August-1978	2,787	1,907	1,526	47,403	189,195
September-1978	2,780	1,895	1,516	47,685	187,478
October-1978	2,777	1,724	1,380	47,282	181,387
November-1978	2,777	1,754	1,404	47,821	186,627
December-1978	2,767	1,762	1,411	47,385	189,628
January-1979	2,753	1,803	1,443	48,662	190,752
February-1979	2,752	1,768	1,415	49,128	186,524
March-1979	2,761	1,818	1,455	50,693	195,735
April-1979	2,758	1,810	1,448	51,026	197,443
May-1979	2,769	1,802	1,442	50,879	193,419
June-1979	2,764	1,821	1,457	51,471	200,612
July-1979	2,753	1,816	1,453	53,057	208,264
August-1979	2,745	1,839	1,472	54,450	215,254
September-1979	2,739	1,815	1,452	55,220	219,713
October-1979	2,740	1,753	1,403	52,604	210,183
November-1979	2,739	1,779	1,424	54,503	221,857
December-1979	2,739	1,789	1,432	56,856	228,882
January-1980	2,738	1,815	1,452	59,693	230,340
February-1980	2,737	1,781	1,425	60,074	232,028
March-1980	2,728	1,673	1,339	56,808	218,453
April-1980	2,727	1,694	1,356	58,158	226,890
May-1980	2,734	1,719	1,376	60,146	238,050
June-1980	2,749	1,742	1,394	60,581	241,717
July-1980	2,762	1,787	1,430	62,578	247,373
August-1980	2,761	1,809	1,448	65,029	255,070
September-1980	2,754	1,819	1,456	63,232	255,930
October-1980	2,753	1,823	1,459	65,771	255,569
November-1980	2,748	1,834	1,468	67,970	263,619
December-1980	2,759	1,806	1,445	66,981	265,648

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months [−18, −6] to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1981	2,778	1,814	1,452	66,811	261,618
February-1981	2,774	1,815	1,452	67,062	261,526
March-1981	2,788	1,840	1,472	68,531	276,266
April-1981	2,788	1,850	1,480	70,058	277,539
May-1981	2,793	1,856	1,485	70,021	283,476
June-1981	2,804	1,847	1,478	71,320	280,811
July-1981	2,809	1,829	1,464	69,877	277,357
August-1981	2,954	1,810	1,448	67,863	261,935
September-1981	2,949	1,759	1,408	65,402	255,905
October-1981	2,949	1,806	1,445	66,041	255,782
November-1981	2,936	1,799	1,440	68,677	263,351
December-1981	2,919	1,786	1,429	67,074	258,255
January-1982	2,934	1,772	1,418	64,826	256,637
February-1982	2,920	1,732	1,386	64,803	249,571
March-1982	2,905	1,709	1,368	65,537	252,515
April-1982	2,905	1,722	1,378	67,803	257,076
May-1982	2,893	1,695	1,356	67,605	256,595
June-1982	2,886	1,671	1,337	67,084	255,345
July-1982	2,887	1,664	1,332	66,416	249,186
August-1982	2,876	1,688	1,351	66,749	265,166
September-1982	2,872	1,692	1,354	68,888	270,708
October-1982	2,859	1,755	1,404	71,993	289,404
November-1982	2,858	1,805	1,444	71,316	297,725
December-1982	2,849	1,799	1,440	71,396	295,346
January-1983	2,835	1,831	1,465	73,305	290,114
February-1983	2,830	1,863	1,491	73,584	294,856
March-1983	2,829	1,881	1,505	75,618	298,545
April-1983	2,832	1,908	1,527	76,768	308,700
May-1983	2,838	1,966	1,573	77,088	309,354
June-1983	2,843	1,974	1,580	79,328	321,485
July-1983	2,839	1,957	1,566	81,135	323,362
August-1983	2,838	1,946	1,557	79,501	320,425
September-1983	2,854	1,943	1,555	80,886	336,958
October-1983	2,861	1,915	1,532	79,515	323,366
November-1983	2,861	1,929	1,544	82,669	331,708
December-1983	2,853	1,916	1,533	81,679	336,259

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18,\ -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1983	2,835	1,831	1,465	73,305	290,114
February-1983	2,830	1,863	1,491	73,584	294,856
March-1983	2,829	1,881	1,505	75,618	298,545
April-1983	2,832	1,908	1,527	76,768	308,700
May-1983	2,838	1,966	1,573	77,088	309,354
June-1983	2,843	1,974	1,580	79,328	321,485
July-1983	2,839	1,957	1,566	81,135	323,362
August-1983	2,838	1,946	1,557	79,501	320,425
September-1983	2,854	1,943	1,555	80,886	336,958
October-1983	2,861	1,915	1,532	79,515	323,366
November-1983	2,861	1,929	1,544	82,669	331,708
December-1983	2,853	1,916	1,533	81,679	336,259
January-1984	2,853	1,908	1,527	82,418	329,745
February-1984	2,841	1,879	1,504	79,131	314,761
March-1984	2,843	1,880	1,504	80,146	320,934
April-1984	2,840	1,862	1,490	81,276	320,667
May-1984	2,850	1,837	1,470	80,528	315,956
June-1984	2,843	1,829	1,464	81,811	322,322
July-1984	2,835	1,808	1,447	80,441	320,420
August-1984	2,826	1,836	1,469	84,131	347,865
September-1984	2,824	1,821	1,457	85,008	352,286
October-1984	2,810	1,793	1,435	85,722	352,087
November-1984	2,800	1,771	1,417	84,123	348,478
December-1984	2,806	1,773	1,419	85,984	354,411
January-1985	2,806	1,824	1,460	86,736	372,393
February-1985	2,798	1,823	1,459	87,297	377,028
March-1985	2,805	1,820	1,456	85,638	371,961
April-1985	2,797	1,807	1,446	85,850	382,071
May-1985	2,778	1,806	1,445	86,638	390,781
June-1985	2,775	1,800	1,440	88,129	407,849
July-1985	2,776	1,810	1,448	87,304	398,066
August-1985	2,771	1,805	1,444	84,981	399,759
September-1985	2,776	1,774	1,420	84,416	393,679
October-1985	2,790	1,775	1,420	85,547	406,800
November-1985	2,801	1,796	1,437	87,146	423,115
December-1985	2,811	1,808	1,447	90,760	424,310

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18,\ -6]$ to be eligible. Next a real dollar minimum is applied (\$110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in \$ millions.

Month End	Eligible Stocks	After \$ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1986	2,814	1,811	1,449	89,155	432,331
February-1986	2,815	1,848	1,479	89,559	446,721
March-1986	2,827	1,875	1,500	89,193	462,258
April-1986	2,850	1,866	1,493	89,960	457,179
May-1986	2,852	1,873	1,499	91,908	485,149
June-1986	2,872	1,872	1,498	91,315	480,470
July-1986	2,891	1,845	1,476	87,601	458,038
August-1986	2,895	1,851	1,481	87,435	476,741
September-1986	2,892	1,819	1,456	87,450	461,657
October-1986	2,888	1,823	1,459	87,937	458,894
November-1986	2,888	1,816	1,453	89,658	455,387
December-1986	2,864	1,786	1,429	88,225	456,993
January-1987	2,859	1,817	1,454	92,972	480,443
February-1987	2,859	1,848	1,479	92,862	491,320
March-1987	2,846	1,848	1,479	94,787	501,792
April-1987	2,842	1,832	1,466	91,896	480,356
May-1987	2,834	1,843	1,475	91,740	480,764
June-1987	2,826	1,845	1,476	93,004	506,244
July-1987	2,825	1,851	1,481	96,004	514,715
August-1987	2,816	1,839	1,472	97,943	529,616
September-1987	2,811	1,842	1,474	96,263	526,935
October-1987	2,798	1,663	1,331	86,883	448,550
November-1987	2,807	1,621	1,297	89,550	448,411
December-1987	2,816	1,637	1,310	93,713	467,014
January-1988	2,813	1,661	1,329	93,162	475,180
February-1988	2,806	1,695	1,356	91,044	487,994
March-1988	2,831	1,735	1,388	91,263	482,851
April-1988	2,833	1,720	1,376	92,701	492,393
May-1988	2,830	1,700	1,360	92,523	485,133
June-1988	2,865	1,733	1,387	97,860	500,714
July-1988	2,886	1,726	1,381	98,294	479,554
August-1988	2,925	1,733	1,387	95,282	460,562
September-1988	2,948	1,738	1,391	98,840	484,219
October-1988	2,975	1,735	1,388	97,203	487,318
November-1988	2,988	1,707	1,366	96,073	487,092
December-1988	3,022	1,724	1,380	97,646	475,925

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18, -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1989	3,029	1,737	1,390	98,225	499,012
February-1989	3,037	1,741	1,393	99,289	501,429
March-1989	3,044	1,761	1,409	98,432	499,230
April-1989	3,044	1,751	1,401	101,873	521,222
May-1989	3,065	1,777	1,422	104,145	529,630
June-1989	3,068	1,772	1,418	104,040	521,948
July-1989	3,066	1,782	1,426	103,250	542,209
August-1989	3,065	1,779	1,424	103,776	555,294
September-1989	3,057	1,773	1,419	105,665	539,674
October-1989	3,042	1,741	1,393	105,842	523,322
November-1989	3,025	1,730	1,384	108,324	534,105
December-1989	3,025	1,725	1,380	109,762	530,173
January-1990	3,017	1,673	1,339	106,099	515,912
February-1990	3,006	1,675	1,340	109,221	509,470
March-1990	3,004	1,691	1,353	108,373	514,046
April-1990	3,006	1,677	1,342	107,403	501,258
May-1990	3,004	1,709	1,368	107,991	514,933
June-1990	3,011	1,715	1,372	107,398	522,208
July-1990	3,010	1,690	1,352	109,436	515,513
August-1990	3,012	1,618	1,296	104,207	486,391
September-1990	3,019	1,572	1,258	105,375	476,382
October-1990	3,022	1,536	1,229	102,000	469,415
November-1990	3,029	1,556	1,245	106,422	499,758
December-1990	3,042	1,562	1,250	110,196	526,850
January-1991	3,041	1,590	1,272	113,346	543,114
February-1991	3,046	1,650	1,320	112,945	548,497
March-1991	3,057	1,688	1,351	113,080	546,596
April-1991	3,071	1,698	1,359	110,738	546,402
May-1991	3,076	1,712	1,370	114,361	575,570
June-1991	3,121	1,713	1,371	111,298	558,845
July-1991	3,140	1,738	1,391	110,195	557,589
August-1991	3,159	1,757	1,406	116,109	575,865
September-1991	3,182	1,770	1,416	114,983	573,093
October-1991	3,201	1,793	1,435	114,487	570,483
November-1991	3,224	1,790	1,432	112,594	550,078
December-1991	3,236	1,834	1,468	115,132	575,424

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18,\ -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1992	3,251	1,903	1,523	113,966	567,624
February-1992	3,264	1,928	1,543	114,452	578,987
March-1992	3,289	1,932	1,546	112,271	558,910
April-1992	3,290	1,907	1,526	111,915	557,105
May-1992	3,298	1,921	1,537	113,120	553,860
June-1992	3,301	1,905	1,524	111,176	550,902
July-1992	3,311	1,929	1,544	113,119	559,624
August-1992	3,324	1,920	1,536	111,569	555,116
September-1992	3,346	1,933	1,547	111,230	573,693
October-1992	3,342	1,949	1,560	111,117	570,888
November-1992	3,342	1,984	1,588	113,396	593,931
December-1992	3,340	2,007	1,606	113,646	601,934
January-1993	3,333	2,037	1,630	112,978	577,949
February-1993	3,340	2,017	1,614	116,219	596,701
March-1993	3,326	2,034	1,628	115,848	601,188
April-1993	3,330	2,011	1,609	116,790	605,492
May-1993	3,329	2,031	1,625	117,528	601,322
June-1993	3,338	2,045	1,636	119,469	632,551
July-1993	3,332	2,052	1,642	120,292	620,805
August-1993	3,330	2,072	1,658	121,472	634,396
September-1993	3,342	2,096	1,677	120,647	630,192
October-1993	3,352	2,137	1,710	115,020	611,060
November-1993	3,383	2,124	1,700	117,160	615,668
December-1993	3,379	2,112	1,690	122,210	651,397
January-1994	3,379	2,163	1,731	117,728	631,120
February-1994	3,399	2,164	1,732	118,100	635,642
March-1994	3,393	2,139	1,712	117,392	610,246
April-1994	3,394	2,117	1,694	121,911	625,030
May-1994	3,613	2,198	1,759	112,361	576,592
June-1994	3,621	2,188	1,751	112,568	577,634
July-1994	3,634	2,205	1,764	111,037	571,058
August-1994	3,632	2,205	1,764	113,445	608,629
September-1994	3,630	2,215	1,772	114,095	595,073
October-1994	3,630	2,214	1,772	114,985	587,746
November-1994	3,641	2,198	1,759	115,605	565,412
December-1994	3,634	2,174	1,740	117,248	592,279

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18, -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1995	3,629	2,190	1,752	116,968	577,520
February-1995	3,627	2,209	1,768	115,998	600,214
March-1995	3,623	2,219	1,776	118,425	603,738
April-1995	3,640	2,244	1,796	119,144	603,190
May-1995	3,643	2,264	1,812	118,035	598,975
June-1995	3,649	2,297	1,838	123,156	614,804
July-1995	3,656	2,321	1,857	125,954	620,672
August-1995	3,656	2,348	1,879	125,100	636,805
September-1995	3,646	2,368	1,895	123,911	638,191
October-1995	3,631	2,324	1,860	121,755	621,908
November-1995	3,616	2,312	1,850	126,588	655,911
December-1995	3,608	2,311	1,849	129,213	656,928
January-1996	3,580	2,304	1,844	127,292	656,628
February-1996	3,562	2,301	1,841	129,587	678,347
March-1996	3,559	2,326	1,861	127,195	661,143
April-1996	3,572	2,373	1,899	127,529	666,109
May-1996	3,580	2,421	1,937	128,628	673,644
June-1996	3,611	2,415	1,932	124,823	658,060
July-1996	3,625	2,384	1,908	124,903	637,578
August-1996	3,631	2,403	1,923	127,721	665,696
September-1996	3,633	2,416	1,933	129,880	675,144
October-1996	3,650	2,423	1,939	133,673	667,390
November-1996	3,670	2,444	1,956	135,281	703,688
December-1996	3,673	2,443	1,955	136,924	715,932
January-1997	3,673	2,487	1,990	135,681	712,650
February-1997	3,678	2,465	1,972	135,401	706,475
March-1997	3,727	2,471	1,977	134,063	701,523
April-1997	3,735	2,447	1,958	132,954	719,686
May-1997	3,743	2,520	2,016	134,228	745,841
June-1997	3,747	2,555	2,044	135,435	768,108
July-1997	3,733	2,575	2,060	136,656	784,753
August-1997	3,729	2,592	2,074	138,231	782,523
September-1997	3,727	2,653	2,123	141,863	796,048
October-1997	3,729	2,638	2,111	143,225	784,222
November-1997	3,728	2,602	2,082	144,167	782,661
December-1997	3,717	2,588	2,071	139,707	803,730

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18, -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-1998	3,704	2,565	2,052	138,500	799,835
February-1998	3,709	2,610	2,088	140,044	826,119
March-1998	3,715	2,657	2,126	140,649	824,276
April-1998	3,726	2,676	2,141	140,056	817,180
May-1998	3,716	2,647	2,118	140,132	798,833
June-1998	3,721	2,610	2,088	142,238	785,022
July-1998	3,719	2,570	2,056	138,255	740,887
August-1998	3,720	2,415	1,932	132,368	691,814
September-1998	3,724	2,443	1,955	133,194	716,236
October-1998	3,723	2,447	1,958	133,870	732,472
November-1998	3,758	2,499	2,000	135,352	731,157
December-1998	3,779	2,505	2,004	137,484	748,786
January-1999	3,777	2,548	2,039	134,115	711,461
February-1999	3,781	2,506	2,005	132,408	678,461
March-1999	3,784	2,455	1,964	131,195	714,592
April-1999	3,784	2,494	1,996	142,520	741,320
May-1999	3,787	2,536	2,029	138,406	743,082
June-1999	3,774	2,541	2,033	136,391	766,721
July-1999	3,748	2,521	2,017	138,873	745,749
August-1999	3,737	2,485	1,988	136,969	745,125
September-1999	3,712	2,447	1,958	138,574	754,150
October-1999	3,705	2,426	1,941	137,008	730,662
November-1999	3,687	2,442	1,954	138,180	753,131
December-1999	3,681	2,479	1,984	137,286	775,016
January-2000	3,666	2,480	1,984	138,661	729,816
February-2000	3,664	2,496	1,997	147,239	761,756
March-2000	3,651	2,487	1,990	146,980	837,646
April-2000	3,650	2,427	1,942	138,548	834,972
May-2000	3,637	2,350	1,880	140,208	858,083
June-2000	3,646	2,386	1,909	141,869	856,196
July-2000	3,645	2,364	1,892	140,958	843,598
August-2000	3,628	2,369	1,896	147,504	914,515
September-2000	3,605	2,330	1,864	148,376	909,018
October-2000	3,613	2,308	1,847	141,839	889,167
November-2000	3,606	2,223	1,779	145,632	897,480
December-2000	3,608	2,234	1,788	143,353	975,016

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months [−18, −6] to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-2001	3,583	2,304	1,844	147,903	957,634
February-2001	3,582	2,274	1,820	150,412	935,814
March-2001	3,585	2,263	1,811	149,481	903,586
April-2001	3,585	2,273	1,819	158,111	990,158
May-2001	3,597	2,332	1,866	171,496	959,358
June-2001	3,607	2,343	1,875	172,856	989,909
July-2001	3,609	2,352	1,882	169,456	943,538
August-2001	3,593	2,332	1,866	169,578	917,605
September-2001	3,603	2,256	1,805	159,800	838,540
October-2001	3,621	2,304	1,844	161,772	876,072
November-2001	3,624	2,356	1,885	162,815	904,080
December-2001	3,624	2,382	1,906	169,464	955,859
January-2002	3,624	2,408	1,927	162,954	910,238
February-2002	3,618	2,382	1,906	164,107	905,663
March-2002	3,602	2,425	1,940	169,048	958,823
April-2002	3,601	2,428	1,943	172,130	960,196
May-2002	3,557	2,417	1,934	164,287	943,440
June-2002	3,570	2,371	1,897	178,449	927,494
July-2002	3,582	2,305	1,844	161,529	834,056
August-2002	3,578	2,301	1,841	163,584	849,370
September-2002	3,581	2,250	1,800	159,712	811,752
October-2002	3,588	2,295	1,836	158,529	839,902
November-2002	3,594	2,358	1,887	159,926	847,470
December-2002	3,591	2,339	1,872	157,227	823,775
January-2003	3,588	2,334	1,868	153,143	800,345
February-2003	3,581	2,323	1,859	153,502	769,355
March-2003	3,572	2,321	1,857	157,723	790,364
April-2003	3,577	2,386	1,909	158,653	839,884
May-2003	3,585	2,476	1,981	161,159	870,741
June-2003	3,598	2,517	2,014	166,342	848,759
July-2003	3,612	2,574	2,060	173,601	862,115
August-2003	3,600	2,593	2,075	173,272	887,960
September-2003	3,586	2,609	2,088	170,584	837,076
October-2003	3,569	2,650	2,120	176,368	894,627
November-2003	3,558	2,682	2,146	176,704	897,929
December-2003	3,542	2,687	2,150	179,900	916,758

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months [-18, -6] to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-2004	3,541	2,740	2,192	179,220	923,592
February-2004	3,546	2,749	2,200	181,157	940,672
March-2004	3,547	2,751	2,201	185,698	950,296
April-2004	3,546	2,739	2,192	180,830	937,200
May-2004	3,545	2,745	2,196	185,336	961,805
June-2004	3,549	2,765	2,212	189,941	994,511
July-2004	3,553	2,732	2,186	181,535	955,859
August-2004	3,553	2,747	2,198	173,728	940,951
September-2004	3,554	2,758	2,207	179,771	978,331
October-2004	3,559	2,767	2,214	181,408	995,268
November-2004	3,564	2,800	2,240	188,966	1,071,544
December-2004	3,562	2,843	2,275	189,464	1,063,497
January-2005	3,546	2,822	2,258	185,772	1,030,884
February-2005	3,555	2,829	2,264	185,685	1,061,174
March-2005	3,560	2,821	2,257	182,520	1,059,125
April-2005	3,580	2,794	2,236	176,961	1,017,062
May-2005	3,566	2,790	2,232	188,530	1,069,430
June-2005	3,570	2,819	2,256	188,423	1,110,434
July-2005	3,580	2,856	2,285	193,418	1,134,691
August-2005	3,587	2,853	2,283	195,411	1,115,715
September-2005	3,587	2,844	2,276	198,252	1,139,793
October-2005	3,583	2,826	2,261	196,487	1,106,207
November-2005	3,575	2,837	2,270	198,837	1,127,509
December-2005	3,551	2,828	2,263	199,279	1,110,412
January-2006	3,526	2,839	2,272	206,133	1,206,508
February-2006	3,517	2,851	2,281	203,219	1,195,377
March-2006	3,502	2,862	2,290	207,362	1,235,369
April-2006	3,487	2,857	2,286	205,734	1,236,690
May-2006	3,463	2,814	2,252	203,810	1,194,254
June-2006	3,454	2,792	2,234	201,826	1,196,000
July-2006	3,452	2,771	2,217	203,666	1,169,771
August-2006	3,440	2,766	2,213	206,333	1,215,861
September-2006	3,429	2,771	2,217	206,667	1,215,890
October-2006	3,413	2,774	2,220	213,259	1,271,045
November-2006	3,400	2,775	2,220	215,115	1,306,985
December-2006	3,382	2,765	2,212	216,046	1,309,207

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18, -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-2007	3,360	2,756	2,205	218,251	1,321,867
February-2007	3,348	2,748	2,199	218,295	1,306,914
March-2007	3,330	2,733	2,187	217,785	1,319,870
April-2007	3,311	2,740	2,192	218,210	1,340,112
May-2007	3,287	2,713	2,171	233,618	1,398,424
June-2007	3,271	2,699	2,160	231,180	1,376,766
July-2007	3,254	2,681	2,145	218,737	1,283,589
August-2007	3,225	2,623	2,099	223,274	1,316,534
September-2007	3,198	2,608	2,087	224,636	1,357,727
October-2007	3,184	2,605	2,084	214,001	1,358,045
November-2007	3,168	2,541	2,033	209,332	1,310,035
December-2007	3,150	2,507	2,006	211,724	1,326,367
January-2008	3,145	2,475	1,980	204,239	1,258,913
February-2008	3,128	2,444	1,956	200,126	1,222,064
March-2008	3,112	2,408	1,927	208,298	1,248,193
April-2008	3,095	2,390	1,912	210,008	1,309,163
May-2008	3,084	2,378	1,903	216,576	1,387,309
June-2008	3,073	2,327	1,862	199,039	1,307,111
July-2008	3,065	2,310	1,848	216,988	1,342,931
August-2008	3,059	2,318	1,855	224,046	1,371,713
September-2008	3,049	2,238	1,791	233,707	1,329,074
October-2008	3,032	2,115	1,692	212,932	1,163,097
November-2008	3,035	2,010	1,608	210,014	1,111,979
December-2008	3,022	2,007	1,606	220,841	1,139,157
January-2009	3,016	1,965	1,572	199,068	1,061,132
February-2009	3,009	1,889	1,512	191,200	1,009,453
March-2009	3,007	1,951	1,561	183,776	1,031,365
April-2009	3,001	2,035	1,628	199,611	1,141,281
May-2009	2,958	2,081	1,665	198,635	1,119,698
June-2009	2,967	2,092	1,674	209,327	1,143,122
July-2009	2,970	2,136	1,709	219,484	1,199,247
August-2009	2,969	2,158	1,727	215,295	1,232,221
September-2009	2,966	2,185	1,748	223,788	1,268,425
October-2009	2,966	2,155	1,724	209,745	1,206,184
November-2009	2,953	2,149	1,720	214,711	1,267,479
December-2009	2,958	2,184	1,748	225,862	1,325,731

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18,\ -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-2010	2,953	2,187	1,750	214,347	1,278,329
February-2010	2,953	2,207	1,766	225,754	1,310,871
March-2010	2,950	2,239	1,792	229,015	1,384,292
April-2010	2,937	2,257	1,806	241,728	1,459,300
May-2010	2,931	2,222	1,778	234,210	1,387,485
June-2010	2,945	2,213	1,771	222,428	1,279,107
July-2010	2,938	2,231	1,785	232,487	1,346,238
August-2010	2,944	2,220	1,776	213,067	1,269,173
September-2010	2,935	2,242	1,794	224,592	1,396,989
October-2010	2,943	2,259	1,808	223,543	1,417,621
November-2010	2,938	2,261	1,809	234,494	1,473,691
December-2010	2,931	2,284	1,828	249,250	1,587,723
January-2011	2,922	2,295	1,836	239,983	1,555,299
February-2011	2,919	2,306	1,845	245,926	1,602,757
March-2011	2,917	2,306	1,845	253,751	1,666,237
April-2011	2,910	2,307	1,846	254,658	1,741,632
May-2011	2,910	2,297	1,838	252,909	1,697,074
June-2011	2,898	2,267	1,814	253,454	1,686,626
July-2011	2,894	2,259	1,808	244,622	1,611,230
August-2011	2,893	2,216	1,773	234,161	1,534,980
September-2011	2,899	2,170	1,736	221,434	1,388,354
October-2011	2,900	2,206	1,765	245,684	1,570,742
November-2011	2,905	2,204	1,764	242,240	1,557,998
December-2011	2,905	2,211	1,769	244,470	1,549,758
January-2012	2,901	2,231	1,785	254,813	1,632,537
February-2012	2,895	2,235	1,788	253,567	1,685,201
March-2012	2,895	2,252	1,802	257,676	1,697,825
April-2012	2,900	2,250	1,800	250,638	1,657,550
May-2012	2,914	2,250	1,800	244,552	1,552,961
June-2012	2,912	2,265	1,812	258,833	1,580,176
July-2012	2,907	2,263	1,811	245,435	1,535,162
August-2012	2,914	2,273	1,819	255,035	1,608,822
September-2012	2,896	2,278	1,823	263,508	1,585,743
October-2012	2,893	2,266	1,813	253,707	1,609,798
November-2012	2,898	2,260	1,808	258,939	1,629,728
December-2012	2,895	2,270	1,816	263,836	1,621,796

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18, -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-2013	2,892	2,293	1,835	264,818	1,705,662
February-2013	2,887	2,296	1,837	257,386	1,731,399
March-2013	2,883	2,297	1,838	269,967	1,819,774
April-2013	2,877	2,296	1,837	270,702	1,779,725
May-2013	2,854	2,298	1,839	283,240	1,853,753
June-2013	2,838	2,283	1,827	293,563	1,838,522
July-2013	2,830	2,288	1,831	309,792	1,948,891
August-2013	2,823	2,293	1,835	296,906	1,883,219
September-2013	2,815	2,309	1,848	306,836	1,918,977
October-2013	2,805	2,315	1,852	299,331	1,996,293
November-2013	2,790	2,310	1,848	320,760	2,057,528
December-2013	2,777	2,314	1,852	313,811	2,082,686
January-2014	2,766	2,307	1,846	302,315	2,010,658
February-2014	2,753	2,320	1,856	306,613	2,081,855
March-2014	2,749	2,304	1,844	308,877	2,118,886
April-2014	2,741	2,288	1,831	300,389	2,079,653
May-2014	2,737	2,274	1,820	307,235	2,103,922
June-2014	2,732	2,278	1,823	314,848	2,186,788
July-2014	2,725	2,267	1,814	298,267	2,081,529
August-2014	2,717	2,270	1,816	310,069	2,155,488
September-2014	2,716	2,250	1,800	298,624	2,065,074
October-2014	2,711	2,243	1,795	329,103	2,193,613
November-2014	2,717	2,248	1,799	313,943	2,187,498
December-2014	2,711	2,237	1,790	329,349	2,245,886
January-2015	2,703	2,230	1,784	311,089	2,124,244
February-2015	2,693	2,230	1,784	328,474	2,237,962
March-2015	2,690	2,221	1,777	334,813	2,286,110
April-2015	2,687	2,225	1,780	323,457	2,254,695
May-2015	2,686	2,217	1,774	332,846	2,319,348
June-2015	2,680	2,200	1,760	344,643	2,334,126
July-2015	2,675	2,186	1,749	333,666	2,322,651
August-2015	2,670	2,176	1,741	319,244	2,221,083
September-2015	2,659	2,151	1,721	312,979	2,161,430
October-2015	2,643	2,149	1,720	326,646	2,267,865
November-2015	2,645	2,156	1,725	338,271	2,309,966
December-2015	2,639	2,145	1,716	319,739	2,163,982

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months [−18, −6] to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-2016	2,628	2,102	1,682	304,781	2,072,964
February-2016	2,620	2,091	1,673	300,673	2,077,084
March-2016	2,613	2,108	1,687	313,616	2,196,329
April-2016	2,602	2,122	1,698	313,158	2,215,682
May-2016	2,557	2,092	1,674	313,750	2,259,515
June-2016	2,543	2,071	1,657	317,854	2,172,081
July-2016	2,546	2,086	1,669	330,058	2,309,892
August-2016	2,534	2,084	1,668	338,005	2,377,730
September-2016	2,518	2,075	1,660	345,553	2,388,112
October-2016	2,511	2,060	1,648	333,037	2,333,828
November-2016	2,498	2,067	1,654	371,818	2,547,856
December-2016	2,496	2,074	1,660	386,874	2,561,973
January-2017	2,490	2,079	1,664	365,169	2,570,078
February-2017	2,486	2,065	1,652	372,238	2,621,476
March-2017	2,483	2,068	1,655	371,653	2,635,251
April-2017	2,474	2,060	1,648	377,157	2,709,580
May-2017	2,457	2,045	1,636	366,758	2,695,521
June-2017	2,443	2,038	1,631	388,057	2,753,834
July-2017	2,454	2,048	1,639	372,323	2,728,318
August-2017	2,445	2,030	1,624	367,798	2,687,630
September-2017	2,430	2,040	1,632	382,294	2,794,427
October-2017	2,433	2,030	1,624	394,380	2,873,361
November-2017	2,422	2,034	1,628	412,068	2,956,042
December-2017	2,411	2,029	1,624	402,840	2,962,668
January-2018	2,410	2,039	1,632	390,189	2,971,092
February-2018	2,412	2,032	1,626	375,446	2,816,824
March-2018	2,404	2,019	1,616	399,683	2,861,387
April-2018	2,405	2,023	1,619	401,575	2,842,501
May-2018	2,410	2,034	1,628	430,801	2,978,070
June-2018	2,412	2,038	1,631	433,863	2,962,780
July-2018	2,409	2,034	1,628	433,462	3,077,618
August-2018	2,410	2,037	1,630	438,110	3,186,325
September-2018	2,420	2,032	1,626	441,534	3,150,733
October-2018	2,421	2,024	1,620	402,578	2,909,988
November-2018	2,420	2,009	1,608	418,059	2,985,492
December-2018	2,413	1,980	1,584	380,757	2,706,263

Table IA-1 -Cont'd.
Sample Construction

Stocks must have 60 months non-missing data on CRSP and a book value in CRSP/Compustat in months $[-18,\ -6]$ to be eligible. Next a real dollar minimum is applied ($110 million in December 2021). Finally, the smallest 10% of stocks are removed prior to Nasdaq eligibility, and 20% after Nasdaq stocks enter the sample (after December, 1977). This table shows the effect of each filter on the sample size, as well as the smallest and median sized stocks eligible for the ensuing month in $ millions.

Month End	Eligible Stocks	After $ criterion	Final Sample	Minimum Mkt. Cap.	Median Mkt. Cap.
January-2019	2,424	2,013	1,611	393,212	2,894,347
February-2019	2,433	2,028	1,623	402,867	2,932,791
March-2019	2,452	2,031	1,625	394,455	2,855,955
April-2019	2,464	2,044	1,636	400,243	2,965,515
May-2019	2,468	2,014	1,612	389,269	2,710,687
June-2019	2,474	2,036	1,629	399,359	2,932,450
July-2019	2,484	2,045	1,636	385,297	2,904,489
August-2019	2,483	2,021	1,617	379,908	2,792,264
September-2019	2,480	2,031	1,625	373,952	2,769,918
October-2019	2,476	2,027	1,622	389,818	2,950,736
November-2019	2,490	2,043	1,635	399,210	2,974,486
December-2019	2,479	2,048	1,639	412,608	3,038,905
January-2020	2,480	2,044	1,636	374,938	2,941,826
February-2020	2,482	2,027	1,622	353,533	2,704,327
March-2020	2,479	1,930	1,544	309,687	2,265,374
April-2020	2,483	1,985	1,588	329,175	2,450,341
May-2020	2,463	1,997	1,598	322,659	2,567,540
June-2020	2,468	2,022	1,618	324,918	2,584,908
July-2020	2,481	2,039	1,632	323,619	2,665,451
August-2020	2,484	2,052	1,642	340,131	2,750,091
September-2020	2,482	2,036	1,629	348,337	2,697,722
October-2020	2,489	2,039	1,632	347,535	2,773,594
November-2020	2,492	2,106	1,685	372,456	3,101,724
December-2020	2,488	2,120	1,696	377,784	3,261,970
January-2021	2,481	2,144	1,716	380,913	3,255,209
February-2021	2,476	2,177	1,742	378,138	3,400,956
March-2021	2,474	2,187	1,750	395,257	3,477,867
April-2021	2,469	2,172	1,738	408,353	3,520,544
May-2021	2,477	2,178	1,743	410,111	3,586,850
June-2021	2,477	2,192	1,754	400,369	3,571,701
July-2021	2,484	2,173	1,739	398,008	3,587,144
August-2021	2,484	2,173	1,739	401,794	3,594,588
September-2021	2,481	2,161	1,729	411,103	3,507,657
October-2021	2,480	2,149	1,720	443,766	3,748,788
November-2021	2,477	2,116	1,693	437,320	3,737,703

Table IA-2

Ex-post Out-of-Sample Optimal γ^* and characteristic set:
Sampling properties of certainty equivalent returns
For investor with power utility and coefficient of relative risk aversion, $\gamma = 5$.

Weight tilts (θ) are estimated for 63 characteristic sets under each of 14 values of the loss function curvature (γ^*), using both rolling and updating protocols. Of these 882 cases that with the highest 1%ile value of the out of sample Certainty Equivalent is reported in basis points per month. The characteristic symbols are: ζ: momentum, V: book-to-market ratio, S: log size, β: from lagged 60-month market model, $\overline{r}$: average same-month return over the previous 5 years, σ_ϵ: standard deviation of lagged 60-month market model residual.

	Updating Protocol					Rolling Protocol				
Next	Optimal		Certainty Equivalent			Optimal		Certainty Equivalent		
Year	Chars	γ^*	1%ile	Mean	Std Dev	Chars	γ^*	1%ile	Mean	Std Dev
1990	**VWI**		76.2	84.1	3.3	**VWI**		76.2	84.1	3.3
1990	**EWI**		95.0	99.5	1.9	**EWI**		95.0	99.5	1.9
1990	ζ,V,S,$\overline{r}$,σ_ϵ	5	313.6	372.3	26.1	ζ,V,S,$\overline{r}$,σ_ϵ	5	314.4	373.7	26.6
1991	ζ,V,S,$\overline{r}$,σ_ϵ	5	290.0	345.2	24.3	ζ,V,S,$\overline{r}$,σ_ϵ	5	285.6	342.1	25.5
1992	ζ,V,S,$\overline{r}$,σ_ϵ	5	303.8	357.5	24.1	ζ,V,S,β,$\overline{r}$,σ_ϵ	5	281.5	342.3	27.7
1993	ζ,V,S,$\overline{r}$,σ_ϵ	5	305.4	357.8	23.5	ζ,V,S,β,$\overline{r}$,σ_ϵ	6	270.1	325.6	25.6
1994	ζ,V,S,$\overline{r}$,σ_ϵ	5	307.5	359.2	23.1	ζ,V,S,β,$\overline{r}$,σ_ϵ	6	271.6	327.7	25.9
1995	ζ,V,S,$\overline{r}$,σ_ϵ	5	297.4	347.5	22.4	ζ,V,S,β,$\overline{r}$,σ_ϵ	6	260.4	318.9	26.8
1996	ζ,V,S,β,$\overline{r}$,σ_ϵ	5	291.8	341.9	23.0	ζ,V,S,$\overline{r}$,σ_ϵ	5	288.9	353.8	29.8
1997	ζ,V,S,β,$\overline{r}$,σ_ϵ	5	286.2	334.2	22.1	ζ,V,S,$\overline{r}$,σ_ϵ	5	291.9	358.7	30.2
1998	ζ,V,S,β,$\overline{r}$,σ_ϵ	5	306.4	354.8	22.5	ζ,V,S,$\overline{r}$,σ_ϵ	5	306.2	377.8	33.1
1999	ζ,V,S,β,$\overline{r}$,σ_ϵ	6	279.4	321.5	19.2	ζ,V,$\overline{r}$,σ_ϵ	6	251.0	319.7	30.4
2000	ζ,V,S,β,$\overline{r}$,σ_ϵ	5	267.8	318.7	23.3	ζ,V,$\overline{r}$,σ_ϵ	7	219.7	296.7	32.6
2001	ζ,S,β,$\overline{r}$,σ_ϵ	7	217.5	273.8	21.5	ζ,σ_ϵ	7	110.8	141.7	13.7
2002	ζ,S,β,$\overline{r}$,σ_ϵ	8	188.3	238.6	20.1	$\overline{r}$	22	71.4	93.2	9.2
2003	ζ,S,β,$\overline{r}$,σ_ϵ	8	193.1	243.9	20.3	ζ,V	7	83.0	114.6	14.4
2004	ζ,S,β,$\overline{r}$,σ_ϵ	8	184.2	231.2	18.7	ζ,V	8	76.3	107.3	13.8
2005	ζ,S,β,$\overline{r}$,σ_ϵ	8	183.2	230.2	18.3	ζ,V	8	68.3	98.3	13.2
2006	ζ,S,$\overline{r}$,σ_ϵ	9	181.1	220.5	15.6	ζ,V,	8	100.9	129.8	13.1
2007	ζ,S,β,$\overline{r}$,σ_ϵ	9	177.3	215.1	15.5	ζ,V	8	95.8	124.0	12.9
2008	ζ,S,$\overline{r}$,σ_ϵ	9	166.3	201.8	14.0	ζ,V	8	65.3	90.8	11.1
2009	ζ,S,$\overline{r}$,σ_ϵ	9	157.7	192.6	13.9	**EWI**		13.8	19.0	2.2
2010	ζ,S,$\overline{r}$,σ_ϵ	11	139.9	166.7	11.4	**EWI**		23.4	28.8	2.3
2011	ζ,S,β,$\overline{r}$,σ_ϵ	9	139.4	173.6	14.1	**EWI**		16.5	21.8	2.3
2012	ζ,S,β,$\overline{r}$,σ_ϵ	9	139.1	173.0	14.0	**EWI**		1.0	6.2	2.3
2013	ζ,S,β,$\overline{r}$,σ_ϵ	9	137.9	170.6	13.6	**EWI**		-0.4	1.3	2.3
2014	ζ,S,β,$\overline{r}$,σ_ϵ	9	140.9	173.1	13.3	**EWI**		19.9	25.2	2.3
2015	ζ,S,$\overline{r}$,σ_ϵ	11	137.1	161.3	10.2	**EWI**		16.4	21.5	2.2
2016	ζ,S,$\overline{r}$,σ_ϵ	11	135.7	159.5	10.0	S,$\overline{r}$,σ_ϵ	11	43.9	62.5	8.4
2017	ζ,S,$\overline{r}$,σ_ϵ	11	134.7	157.7	9.6	S,$\overline{r}$,σ_ϵ	12	42.9	56.5	6.2
2018	ζ,S,$\overline{r}$,σ_ϵ	11	133.1	154.8	9.2	S,σ_ϵ	22	54.2	64.3	4.4
2019	ζ,S,$\overline{r}$,σ_ϵ	11	128.8	150.2	9.1	V,S,σ_ϵ	22	41.6	55.2	5.7
2020	ζ,S,$\overline{r}$,σ_ϵ	11	128.3	149.1	8.7	V,σ_ϵ	22	48.9	62.1	5.7
2021	ζ,S,$\overline{r}$,σ_ϵ	11	124.9	145.1	8.5	V,S,σ_ϵ	22	45.2	61.2	6.8

Table IA-3

Optimal γ^* and each characteristic sets:
Sampling properties of certainty equivalent returns
For investor with power utility and coefficient of relative risk aversion, $\gamma = 8$.

Weight tilts (θ) are estimated for 63 characteristic sets under each of 14 values of the loss function curvature (γ^*), using both rolling and updating protocols. Of these 882 cases that with the highest 1%ile value of the out of sample Certainty Equivalent is reported in basis points per month. The characteristic symbols are: ζ: momentum, V: book-to-market ratio, S: log size, β: from lagged 60-month market model, $\bar{r}$: average same-month return over the previous 5 years, σ_ϵ: standard deviation of lagged 60-month market model residual.

	Updating Protocol					Rolling Protocol				
Next	Optimal		Certainty Equivalent			Optimal		Certainty Equivalent		
Year	Chars	γ^*	1%ile	Mean	Std Dev	Chars	γ^*	1%ile	Mean	Std Dev
1990	**VWI**		32.3	41.4	3.9	**VWI**		32.3	41.4	3.9
1990	**EWI**		30.4	36.5	2.6	**EWI**		30.4	36.5	2.6
1990	ζ,V,S,β,$\bar{r}$,σ_ϵ	8	220.7	262.7	18.5	ζ,V,S,β,$\bar{r}$,σ_ϵ	8	219.8	262.2	18.6
1990	ζ,V,S,β,$\bar{r}$,σ_ϵ	9	222.9	260.7	16.8	ζ,V,S,β,$\bar{r}$,σ_ϵ	9	220.3	258.7	17.0
1991	ζ,V,S,β,$\bar{r}$,σ_ϵ	9	203.0	239.2	15.9	ζ,V,S,β,$\bar{r}$,σ_ϵ	9	197.5	234.2	16.6
1992	ζ,V,S,$\bar{r}$,σ_ϵ	8	213.0	252.8	21.3	ζ,V,S,β,$\bar{r}$,σ_ϵ	9	194.3	233.0	17.4
1993	ζ,V,S,$\bar{r}$,σ_ϵ	8	214.4	253.2	17.0	ζ,V,S,β,$\bar{r}$,σ_ϵ	9	183.4	225.6	18.6
1994	ζ,V,S,$\bar{r}$,σ_ϵ	8	215.2	252.8	16.6	ζ,V,S,β,$\bar{r}$,σ_ϵ	9	185.8	227.7	18.6
1995	ζ,V,S,$\bar{r}$,σ_ϵ	8	208.0	244.2	16.0	ζ,V,S,β,$\bar{r}$,σ_ϵ	10	179.0	218.9	17.8
1996	ζ,V,S,$\bar{r}$,σ_ϵ	8	207.1	241.7	15.2	ζ,V,S,β,$\bar{r}$,σ_ϵ	8	204.3	250.1	20.3
1997	ζ,V,S,$\bar{r}$,σ_ϵ	8	205.9	239.0	14.6	ζ,V,S,$\bar{r}$,σ_ϵ	8	208.4	254.5	20.5
1998	ζ,V,S,$\bar{r}$,σ_ϵ	8	221.2	254.2	14.6	ζ,V,S,β,$\bar{r}$,σ_ϵ	8	214.6	264.4	22.2
1999	ζ,V,S,$\bar{r}$,σ_ϵ	9	206.2	235.0	12.8	ζ,V,S,β,$\bar{r}$,σ_ϵ	12	173.7	211.1	16.7
2000	ζ,V,S,$\bar{r}$,σ_ϵ	9	186.2	217.7	13.4	ζ,$\bar{r}$,σ_ϵ	11	125.5	166.6	17.2
2001	ζ,S,σ_ϵ	10	135.0	160.6	11.0	ζ,σ_ϵ	8	52.6	85.6	14.0
2002	ζ,S,σ_ϵ	16	76.4	105.3	11.5	**VWI**		9.4	21.5	5.2
2003	ζ,S,σ_ϵ	16	75.6	103.9	11.2	ζ,V	9	36.3	64.4	12.2
2004	ζ,S,σ_ϵ	16	75.9	102.7	10.7	ζ,V	9	30.2	59.1	12.6
2005	ζ,S,σ_ϵ	16	77.8	104.0	10.4	**EWI**		24.0	29.2	2.3
2006	ζ,S,σ_ϵ	16	77.1	102.6	10.1	ζ,V	9	58.5	86.9	12.2
2007	ζ,S,σ_ϵ	16	78.7	103.5	9.8	ζ,V	9	56.0	84.0	12.1
2008	ζ,S,σ_ϵ	16	74.3	97.9	9.3	ζ,V	9	27.3	53.1	10.8
2009	ζ,S,σ_ϵ	16	63.4	87.1	9.4	**VWI**		-29.0	-15.7	5.5
2010	ζ,S,σ_ϵ	16	56.0	77.4	8.6	**VWI**		-24.7	-11.0	5.8
2011	ζ,S,σ_ϵ	16	55.9	76.9	8.4	**VWI**		-38.4	-25.0	5.8
2012	ζ,S,σ_ϵ	16	55.8	76.6	8.3	**VWI**		-51.0	-37.7	5.7
2013	ζ,S,σ_ϵ	16	56.7	76.9	8.1	**VWI**		-56.1	-42.6	5.7
2014	ζ,S,σ_ϵ	16	61.0	80.6	7.9	**EWI**		-35.3	-29.0	2.7
2015	ζ,S,σ_ϵ	16	61.0	80.0	7.7	**EWI**		-38.3	-32.3	2.6
2016	ζ,S,σ_ϵ	16	60.5	79.4	7.5	S,β,$\bar{r}$	13	11.8	26.7	6.5
2017	ζ,S,σ_ϵ	16	62.4	80.9	7.4	V,S,σ_ϵ	16	16.7	32.3	6.5
2018	ζ,S,σ_ϵ	16	63.6	81.8	7.2	V,S,σ_ϵ	22	30.4	45.0	6.0
2019	ζ,S,σ_ϵ	16	60.7	78.3	7.1	V,σ_ϵ	22	20.7	34.2	5.8
2020	ζ,S,σ_ϵ	16	63.2	80.5	6.9	V,σ_ϵ	22	28.9	43.1	6.1
2021	ζ,S,σ_ϵ	16	59.0	75.7	6.7	V,σ_ϵ	22	21.3	37.9	7.1

Table IA-4

Sampling properties of out-of-sample Portfolio Return Statistics: First Subperiod

Sampling properties of dynamic optimal PPPs. Portfolio characteristic tilts from the best out-of-sampling performer over the relevant preceding period (shown in Table 1) each year. $\mathcal{CE}_k$ is the certainty equivalent return in basis points per month for a power utility investor with coefficient of relative risk aversion $(\gamma) = k$. $E(r)$, σ, Median, IQR, and MIN are the mean monthly return, the standard deviation of monthly returns, the median monthly return, the interquartile range of monthly returns, and the minimum monthly return–all expressed in basis points per month.
SKEW and KURT are the return skewness and kurtosis measures, and SR is the Sharpe ratio.
Results are for the first 9-year out-of-sample subperiod (1990 – 1998).

Panel A: Benchmark portfolios

	First 9-year subperiod – VWI							First 9-year subperiod – EWI						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_2$	128.8	4.0	121.0	126.1	128.8	131.5	136.6	109.4	2.3	104.8	107.9	109.4	110.9	113.9
$\mathcal{CE}_5$	104.1	4.1	96.1	101.4	104.2	107.0	112.3	77.6	2.4	72.9	75.9	77.6	79.2	82.2
$\mathcal{CE}_8$	77.2	4.5	68.3	74.1	77.2	80.3	86.1	41.3	2.7	36.2	39.5	41.4	43.2	46.4
$E(r)$	144.2	4.0	136.2	141.5	144.2	146.9	152.0	128.6	2.3	124.1	127.1	128.6	130.1	133.1
σ	388.6	4.6	379.6	385.5	388.6	391.7	397.7	430.0	2.3	425.1	428.3	430.0	431.6	434.9
Median	167.4	11.6	144.9	159.3	167.6	175.3	189.7	187.5	9.8	168.1	180.0	187.6	194.1	206.5
IQR	469.9	22.4	426.6	454.5	469.8	484.9	514.7	508.7	17.5	474.6	496.7	508.7	520.5	543.3
MIN	-1,482.9	59.1	-1,602.5	-1,521.9	-1,481.9	-1,442.2	-1,368.6	-1,747.3	28.2	-1,803.2	-1,766.3	-1,747.3	-1,728.2	-1,691.9
SKEW	-6.0	2.9	-11.6	-8.0	-6.0	-4.0	-0.4	-13.7	2.2	-18.0	-15.2	-13.7	-12.2	-9.3
KURT	26.4	4.6	17.4	23.3	26.4	29.4	35.6	35.2	2.5	30.3	33.5	35.2	36.9	40.2
SR	0.9337	0.0364	0.8627	0.9095	0.9337	0.9581	1.0051	0.7158	0.0187	0.6787	0.7034	0.7159	0.7284	0.7520

Table IA-4 –2–

Sampling properties of out-of-sample Portfolio Return Statistics: First Subperiod –2–

Sampling properties of dynamic optimal PPPs. Portfolio characteristic tilts from the best out-of-sampling performer over the relevant preceding period (shown in Table 1) each year. $\mathcal{CE}_k$ is the certainty equivalent return in basis points per month for a power utility investor with coefficient of relative risk aversion $(\gamma) = k$. $E(r)$, σ, Median, IQR, and MIN are the mean monthly return, the standard deviation of monthly returns, the median monthly return, the interquartile range of monthly returns, and the minimum monthly return–all expressed in basis points per month.
SKEW and KURT are the return skewness and kurtosis measures, and SR is the Sharpe ratio.
Results are for the first 9-year out-of-sample subperiod (1990 – 1998).

Panel B: γ 5 loss function

	First subperiod (1990 - 1998) – Dyn. Opt. Updating							First subperiod (1990 - 1998) – Dyn. Opt. Rolling						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_5$	250.1	36.3	173.8	228.8	251.6	274.8	317.8	214.6	83.7	20.7	175.9	224.4	267.2	338.1
$E(r)$	431.0	34.1	365.5	407.6	429.9	453.9	499.1	580.5	50.6	485.2	545.7	579.0	613.5	682.6
σ	831.8	53.3	732.9	794.2	829.9	867.8	938.5	1,185.3	74.5	1,046.9	1,133.3	1,182.4	1,233.1	1,340.5
Median	433.1	53.6	331.9	397.2	432.3	467.7	540.6	522.3	74.3	378.6	472.0	521.6	571.5	671.9
IQR	1,003.6	103.3	810.3	932.1	999.8	1,072.9	1,211.6	1,473.6	139.0	1,211.8	1,378.0	1,470.0	1,566.2	1,757.8
MIN	-2,557.6	452.3	-3,483.6	-2,852.2	-2,546.9	-2,242.3	-1,710.2	-3,175.5	727.9	-4,707.2	-3,662.6	-3,143.7	-2,640.3	-1,907.7
SKEW	-0.2	5.2	-10.3	-3.7	-0.3	3.2	10.0	4.9	5.4	-6.1	1.3	5.0	8.5	15.4
KURT	22.5	13.2	-2.3	13.6	22.2	31.3	49.3	16.9	14.0	-9.8	7.3	16.6	26.2	45.1
SR	1.6441	0.1318	1.3880	1.5533	1.6440	1.7320	1.9068	1.5950	0.1317	1.3340	1.5057	1.5940	1.6819	1.8545

Panel C: γ 8 loss function

	First subperiod (1990 - 1998) – Dyn. Opt. Updating							First subperiod (1990 - 1998) – Dyn. Opt. Rolling						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_8$	182.2	25.2	129.1	166.7	183.5	199.5	227.7	156.1	47.5	48.1	131.3	160.8	188.0	232.3
$E(r)$	320.3	22.8	278.1	304.8	319.5	335.4	366.8	400.5	33.3	338.5	377.4	399.6	422.0	469.8
σ	576.1	37.8	506.2	550.2	574.7	600.5	653.7	780.3	47.9	691.6	746.9	778.4	810.6	879.9
Median	315.3	36.0	247.0	290.2	314.4	339.0	389.0	354.8	48.0	264.4	322.1	353.4	386.4	452.6
IQR	702.1	68.6	575.7	653.7	699.5	747.8	839.9	964.1	87.6	797.2	904.7	963.3	1,021.6	1,141.6
MIN	-1,749.4	309.3	-2,379.6	-1,950.6	-1,738.4	-1,541.0	-1,165.9	-2,040.7	470.7	-3,046.9	-2,353.3	-2,019.0	-1,694.4	-1211.3
SKEW	-0.9	5.0	-8.9	-2.6	0.9	4.3	10.8	5.8	5.2	-4.6	2.3	5.8	9.4	15.8
KURT	21.3	11.7	-1.3	13.3	21.1	29.0	45.0	18.8	14.1	-8.0	8.4	17.3	27.1	47.1
SR	1.7007	0.1266	1.4550	1.6135	1.6991	1.7849	1.9503	1.6153	0.1318	1.3634	1.5236	1.6150	1.7026	1.8793

Table IA-5

Sampling properties of out-of-sample Portfolio Return Statistics: Second Subperiod

Sampling properties of dynamic optimal PPPs. Portfolio characteristic tilts from the best out-of-sampling performer over the relevant preceding period (shown in Table 1) each year. $\mathcal{CE}_k$ is the certainty equivalent return in basis points per month for a power utility investor with coefficient of relative risk aversion $(\gamma) = k$. $E(r)$, σ, Median, IQR, and MIN are the mean monthly return, the standard deviation of monthly returns, the median monthly return, the interquartile range of monthly returns, and the minimum monthly return–all expressed in basis points per month.
SKEW and KURT are the return skewness and kurtosis measures, and SR is the Sharpe ratio.
Results are for the second 23-year out-of-sample subperiod (1999 – 2021).

Panel A: Benchmark portfolios

	Second subperiod (1999 - 2021) – VWI							Second subperiod (1999 - 2021) – EWI						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_2$	60.0	3.5	53.1	57.6	60.0	62.4	66.9	76.6	1.7	73.4	75.4	76.6	77.7	79.8
$\mathcal{CE}_5$	29.8	3.8	22.4	27.3	29.9	32.4	37.4	27.8	1.8	24.3	26.6	27.8	29.0	31.2
$\mathcal{CE}_8$	-2.5	4.2	-10.6	-5.3	-2.5	0.2	5.7	-26.9	2.2	-31.2	-28.4	-26.9	-25.3	-22.61
$E(r)$	79.2	3.5	72.4	76.8	79.2	81.6	85.9	106.8	1.7	103.6	105.6	106.8	107.9	110.0
σ	433.3	4.2	425.2	430.4	433.3	436.3	441.6	542.7	2.1	538.6	541.3	542.7	544.1	546.8
Median	124.0	8.0	107.9	118.6	124.1	129.4	139.4	141.9	6.7	128.8	137.4	141.9	146.4	155.3
IQR	511.2	13.9	484.6	501.8	510.9	520.6	538.8	655.0	12.1	612.0	626.8	635.0	642.9	659.3
MIN	-1,667.4	78.6	-1,819.4	-1,720.6	-1,667.6	-1,613.6	-1,514.3	-2119.7	42.4	-2,206.5	-2,147.9	-2,118.6	-2,090.3	-2,039.7
SKEW	-10.3	1.8	-13.9	-11.5	-10.4	-9.1	-6.8	-6.5	1.2	-8.9	-7.3	-6.5	-5.6	-4.1
KURT	27.9	3.8	20.6	25.3	27.9	30.4	35.4	32.5	1.5	29.5	31.5	32.5	33.5	35.5
SR	0.5245	0.0286	0.4687	0.5052	0.5246	0.5436	0.5807	0.5959	0.0105	0.5756	0.5887	0.5957	0.6029	0.6165

Table IA-5 –2–

Sampling properties of out-of-sample Portfolio Return Statistics: Second Subperiod –2–

Panel B: γ 5 loss function

	Second subperiod (1990 - 2021) – Dyn. Opt. Updating							Second subperiod (1990 - 2021) – Dyn. Opt. Rolling						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_5$	-27.1	89.8	-212.6	-38.6	-8.4	12.0	41.3	-334.0	668.7	-1,690.6	-295.0	-172.2	-113.1	-52.8
$E(r)$	142.6	13.2	116.6	133.8	142.6	151.4	168.5	104.0	13.7	78.1	94.7	103.9	113.0	131.4
σ	717.4	35.7	652.8	692.2	714.9	739.5	794.5	841.1	52.3	748.0	805.0	837.5	873.6	953.9
Median	169.6	20.0	131.0	156.1	169.3	183.0	209.7	122.9	15.4	93.6	112.4	122.4	133.2	154.2
IQR	746.3	36.2	676.5	721.5	746.0	770.5	818.7	688.3	27.4	635.8	669.6	688.3	706.8	742.4
MIN	-3,575.9	899.6	-5,689.1	-4,118.0	-3,390.7	-2,900.2	-2,283.4	-5,002.7	1,112.2	-7,586.0	-5,654.3	-4,799.2	-4,161.0	-3,408.4
SKEW	-3.8	2.7	-9.1	-5.6	-3.8	-2.0	1.6	-2.3	2.8	-6.2	-3.6	-2.2	-0.9	1.6
KURT	74.0	13.0	49.4	65.3	73.7	82.6	100.4	120.4	2.0	93.9	110.9	120.2	129.7	148.2
SR	0.6273	0.0726	0.4844	0.5785	0.6268	0.6756	0.7692	0.3753	0.0611	0.2588	0.3336	0.3750	0.4165	0.4982

Panel C: γ 8 loss function

	Second subperiod (1999 - 2021) – Dyn. Opt. Updating							Second subperiod (1990 - 2021) – Dyn. Opt. Rolling						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_8$	-122.5	75.0	-307.2	-146.7	-105.8	-76.4	-36.4	-265.8	172.2	-687.0	-301.3	-224.7	-173.0	-110.4
$E(r)$	111.7	7.8	96.5	106.4	111.7	116.9	127.1	70.4	8.6	53.9	64.6	70.4	76.1	87.0
σ	583.6	24.2	538.1	567.3	583.0	599.1	633.9	653.0	30.8	597.2	631.5	651.7	672.9	718.0
Median	136.7	12.1	113.4	128.5	136.5	144.9	160.7	107.9	13.1	83.0	99.1	107.7	116.7	134.3
IQR	504.9	24.9	457.0	487.9	505.1	521.6	554.0	631.8	24.4	583.6	615.4	631.6	648.3	679.6
MIN	-3,317.6	497.1	-4,447.6	-3,613.3	-3,254.6	-2,960.9	-2,506.6	-3,783.4	575.4	-5,105.0	-4,107.0	-3,719.0	-3,383.5	-2,836.6
SKEW	-4.3	2.1	-8.5	-5.7	-4.3	-2.9	-0.3	-5.7	208	-9.7	-7.1	-5.7	-4.4	-1.8
KURT	108.7	11.2	87.0	101.2	108.5	116.1	131.2	75.7	11.1	54.9	68.1	75.4	83.1	98.5
SR	0.5853	0.0524	0.4839	0.5500	0.5848	0.6204	0.6898	0.3034	0.0493	0.2087	0.2697	0.3024	0.3363	0.4014

Table IA-6

Sampling properties of out-of-sample Portfolio Performance Statistics
384-month out-of-sample period, 1990 – 2021

Sampling properties of dynamic optimal PPPs. Portfolio characteristic tilts from the best out-of-sampling performer over the relevant preceding period (shown in Table 1) each year. $\mathcal{CE}_k$ is the certainty equivalent return in basis points per month for a power utility investor with coefficient of relative risk aversion $(\gamma) = k$. $E(r)$, σ, Median, IQR, and MIN are the mean monthly return, the standard deviation of monthly returns, the median monthly return, the interquartile range of monthly returns, and the minimum monthly return–all expressed in basis points per month.
SKEW and KURT are the return skewness and kurtosis measures, and SR is the Sharpe ratio.
Results are for the full 32-year out-of-sample subperiod (1990 – 2021).

Panel A: Benchmark portfolios

	VWI							EWI						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_2$	79.3	2.8	73.8	77.4	79.3	81.1	84.6	85.8	1.4	83.1	84.8	85.8	86.7	88.4
$\mathcal{CE}_5$	50.5	2.9	44.6	48.4	50.5	52.4	56.3	41.7	1.4	38.9	40.7	41.7	42.7	44.5
$\mathcal{CE}_8$	19.4	3.3	12.9	17.2	19.3	21.6	25.7	-8.1	1.8	-11.5	-9.3	-8.1	-6.9	-4.6
$E(r)$	97.4	2.8	92.0	95.6	97.4	99.3	102.8	112.9	1.3	110.3	112.0	112.9	113.8	115.6
σ	422.3	3.3	415.7	420.0	422.3	424.5	428.8	513.6	1.7	510.4	512.5	513.6	514.8	517.0
Median	138.3	6.5	125.7	133.9	138.3	142.7	151.1	157.3	6.0	145.4	153.2	157.3	161.3	169.0
IQR	503.7	11.9	480.2	495.8	503.7	511.7	526.7	608.3	10.4	588.2	601.4	608.2	615.4	628.9
MIN	-1,669.1	77.4	-1,824.4	-1,720.9	-1,666.9	-1,614.9	-1,522.1	-2,119.8	42.1	-2,205.9	-2,148.0	-2,118.6	-2,090.6	-2,040.3
SKEW	-31.5	4.7	-40.5	-34.6	-31.6	-28.5	-21.9	-9.9	3.1	-16.1	-12.0	-9.9	-7.8	-4.0
KURT	29.5	3.1	23.6	27.4	29.5	31.6	35.6	33.4	1.3	30.8	32.5	33.4	34.2	36.0
SR	0.6271	0.0234	0.5810	0.6111	0.6270	0.6429	0.6735	0.6196	0.0091	0.6020	0.6134	0.6196	0.6257	0.6375

Panel B: γ 2 loss function

	Dyn. Opt. Updating							Dyn. Opt. Rolling						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_2$	114.8	168.9	33.2	96.7	120.6	142.3	166.5	-7,396.7	2,467.7	-10,000	-10,000	-5,395.9	-4,988.2	-4,943.1
$E(r)$	339.8	28.9	284.1	320.3	338.7	358.9	398.4	379.1	33.2	316.5	356.5	378.4	400.9	447.0
σ	1,424.0	62.9	1,306.0	1,380.7	1,421.0	1,465.2	1,553.3	1,593.5	102.5	1,409.7	1,522.5	1,586.2	1,657.3	1,812.6
Median	346.7	45.3	260.6	315.6	345.9	377.0	437.5	230.4	19.6	189.6	217.9	230.8	243.2	267.8
IQR	1,670.4	94.8	1,492.8	1,606.0	1,667.4	1,731.7	1,863.4	1,014.0	52.5	914.8	978.6	1,012.5	1,047.9	1,121.1
MIN	-5,518.9	911.9	-7,756.8	-5,968.0	-5,374.8	-4,894.4	-4,179.1	-10,086	1,857.3	-14,310	-11,127	-9,861.4	-8,789.2	-7,106.5
SKEW	-0.5	2.7	-5.8	-2.4	-0.5	1.4	4.9	9.3	1.9	5.7	8.1	9.3	10.6	12.9
KURT	35.0	8.9	18.4	28.9	34.8	40.9	52.8	138.8	13.3	113.6	129.6	138.6	147.5	165.8
SR	0.7783	0.0656	0.6479	0.7345	0.7778	0.8219	0.9069	0.7831	0.0702	0.6449	0.7362	0.7832	0.8302	0.9197

Table IA-6 –2–

Sampling properties of out-of-sample Portfolio Performance Statistics –2–

384-month out-of-sample period, 1990 – 2021

Sampling properties of dynamic optimal PPPs. Portfolio characteristic tilts from the best out-of-sampling performer over the relevant preceding period (shown in Table 1) each year. $\mathcal{CE}_k$ is the certainty equivalent return in basis points per month for a power utility investor with coefficient of relative risk aversion $(\gamma) = k$. $E(r)$, σ, Median, IQR, and MIN are the mean monthly return, the standard deviation of monthly returns, the median monthly return, the interquartile range of monthly returns, and the minimum monthly return–all expressed in basis points per month.
SKEW and KURT are the return skewness and kurtosis measures, and SR is the Sharpe ratio.
Results are for the full 32-year out-of-sample subperiod (1990 – 2021).

Panel C: γ 5 loss function

	Dyn. Opt. Updating							Dyn. Opt. Rolling						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_5$	46.7	71.2	-94.3	36.7	60.2	77.1	102.3	-209.0	60.6	-1,325.2	-166.5	-73.6	-26.1	28.6
$E(r)$	223.7	13.7	197.5	214.3	223.6	232.8	250.7	238.0	17.3	204.8	226.2	237.8	249.5	272.6
σ	763.0	32.8	703.9	740.0	761.3	783.8	830.9	975.5	43.2	896.7	945.6	973.7	1,002.6	1,066.2
Median	229.1	19.1	192.1	216.1	229.1	242.0	266.4	196.6	19.3	157.3	183.5	197.1	210.4	232.8
IQR	817.2	36.8	745.6	792.2	816.4	841.2	891.9	835.6	37.0	766.0	809.8	835.6	860.4	910.1
MIN	-3,625.1	857.3	-5,689.1	-4,119.4	-3,421.9	-2,998.3	-2,450.4	-5,032.0	1,089.7	-7,586.0	-5,659.1	-4,825.9	-4,215.3	-3,492.5
SKEW	-0.7	2.4	-5.4	-2.4	-0.7	0.9	4.0	4.2	2.1	0.2	2.9	4.2	5.6	8.3
KURT	57.5	9.4	39.5	51.1	57.3	63.8	76.8	92.5	9.8	73.7	85.9	92.4	98.9	112.4
SR	0.9258	0.0646	0.8000	0.8826	0.9254	0.9698	1.0516	0.7749	0.0606	0.6558	0.7342	0.7750	0.8158	0.8928

Panel D: γ 8 loss function

	Dyn. Opt. Updating							Dyn. Opt. Rolling						
Statistic	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
$\mathcal{CE}_8$	-44.5	58.9	-188.8	-63.0	-31.9	-8.8	22.3	-163.1	144.2	-502.5	-189.7	-130.0	-89.7	-37.8
$E(r)$	170.4	8.5	154.0	164.6	170.4	176.0	187.2	163.3	11.3	141.6	155.3	163.1	170.8	186.0
σ	589.4	20.8	550.2	575.2	588.7	602.9	632.4	707.5	26.6	657.9	689.0	706.9	725.0	761.2
Median	178.1	12.6	153.6	169.5	177.9	186.5	203.0	163.9	15.8	133.8	153.0	163.7	174.5	195.6
IQR	558.7	24.7	511.4	541.6	558.3	575.5	608.2	697.5	26.1	647.6	679.5	696.7	715.1	749.7
MIN	-3317.9	496.7	-4,447.6	-3.613.3	-3,254.7	-2,961.2	-2,508.0	-3,785.2	573.3	-5,105.0	-4,107.0	-3,719.3	-3,386.5	-2,847.5
SKEW	-1.3	2.1	-5.3	-2.7	-1.3	0.1	2.7	-0.1	2.2	-4.5	-1.6	-0.1	1.4	4.1
KURT	80.2	8.5	63.8	74.5	80.1	85.8	97.1	64.0	8.6	47.7	58.0	63.8	69.7	81.3
SR	0.8824	0.0524	0.7807	0.8475	0.8824	0.9181	0.9841	0.7001	0.0549	0.5928	0.6627	0.6997	0.7373	0.8082

Table IA-7

Out-of-Sample 6-factor Fama-French regressions

$$r_{i,t} - r_f = \alpha + \beta_1 \cdot (R_{m,t} - r_f) + \beta_2 \cdot \text{HML} + \beta_3 \cdot \text{SMB} + \beta_4 \cdot \text{MOM} + \beta_5 \cdot \text{RMW} + \beta_6 \cdot \text{CMA} + \epsilon_{i,t}$$

For power utility investor with coefficient of relative risk aversion, $\gamma = 5$. Monthly returns; α in basis points per month.

Panel A. 32-year out-of-sample period: 1990 - 2021

	Updating Protocol							Rolling Protocol						
Coefficient	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
α	98.44	14.92	69.39	88.53	98.20	108.47	127.98	133.80	18.52	98.15	121.16	133.52	146.06	170.84
Mkt	0.57	0.04	0.49	0.54	0.57	0.60	0.65	0.93	0.05	0.83	0.90	0.93	0.96	1.02
HML	1.03	0.07	0.89	0.98	1.03	1.08	1.18	1.18	0.08	1.02	1.12	1.18	1.24	1.35
SMB	-0.02	0.09	-0.20	-0.08	-0.02	0.03	0.14	-0.98	0.10	-1.19	-1.05	-0.98	-0.91	-0.78
MOM	0.63	0.06	0.52	0.60	0.63	0.67	0.75	0.43	0.07	0.30	0.39	0.43	0.48	0.57
RMW	0.58	0.11	0.36	0.51	0.58	0.66	0.80	-0.24	0.14	-0.51	-0.33	-0.24	-0.14	0.04
CMA	0.02	0.10	-0.19	-0.05	0.02	0.09	0.21	0.21	0.12	-0.03	0.13	0.21	0.29	0.44

Panel B. Subperiod 1: 1990 - 1998

	Updating protocol							Rolling protocol						
Coefficient	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
α	166.16	35.41	97.74	141.43	165.73	190.19	236.61	244.99	52.59	144.17	209.62	243.94	278.95	352.57
Mkt	0.03	0.12	-0.21	-0.05	0.03	0.11	0.25	-0.08	0.16	-0.40	-0.19	-0.08	0.03	0.24
HML	2.25	0.25	1.78	2.09	2.25	2.42	2.75	3.49	0.38	2.76	3.22	3.48	3.74	4.27
SMB	0.97	0.16	0.65	0.85	0.96	1.08	1.28	0.45	0.26	-0.05	0.28	0.45	0.63	0.96
MOM	1.69	0.16	1.40	1.59	1.69	1.80	2.01	2.16	0.22	1.74	2.01	2.16	2.31	2.60
RMW	0.71	0.27	0.19	0.53	0.71	0.89	1.25	0.75	0.43	-0.08	0.46	0.75	1.03	1.61
CMA	-0.92	0.32	-1.56	-1.13	-0.91	-0.71	-0.31	-1.98	0.49	-2.97	-2.31	-1.97	-1.65	-1.04

Panel C. Subperiod 2: 1999 - 2021

	Updating protocol							Rolling protocol						
Coefficient	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
α	34.02	14.20	5.90	24.29	33.97	43.66	61.66	16.61	15.22	-12.55	6.20	16.33	26.73	47.19
Mkt	0.75	0.04	0.67	0.72	0.75	0.78	0.83	1.22	0.04	1.14	1.19	1.22	1.25	1.31
HML	0.71	0.07	0.57	0.66	0.70	0.75	0.84	0.63	0.07	0.49	0.58	0.63	0.67	0.76
SMB	-0.08	0.10	-0.27	-0.14	-0.08	-0.01	0.10	-1.00	0.11	-1.23	-1.08	-1.00	-0.92	-0.79
MOM	0.54	0.06	0.41	0.49	0.54	0.58	0.66	0.26	0.07	0.13	0.22	0.26	0.31	0.40
RMW	0.84	0.11	0.62	0.77	0.84	0.92	1.07	0.13	0.14	-0.14	0.04	0.13	0.22	0.40
CMA	0.12	0.11	-0.10	0.05	0.12	0.20	0.34	0.46	0.12	0.23	0.38	0.46	0.54	0.69

Table IA-8

Out-of-Sample 6-factor Fama-French regressions

$$r_{i,t} - r_f = \alpha + \beta_1 \cdot (R_{m,t} - r_f) + \beta_2 \cdot \text{HML} + \beta_3 \cdot \text{SMB} + \beta_4 \cdot \text{MOM} + \beta_5 \cdot \text{RMW} + \beta_6 \cdot \text{CMA} + \epsilon_{i,t}$$

For power utility investor with coefficient of relative risk aversion, $\gamma = 8$. Monthly returns; α in basis points per month.

Panel A. 32-year out-of-sample period: 1990 - 2021

	Updating Protocol							Rolling Protocol						
Coefficient	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
α	44.88	9.20	27.04	38.62	44.86	50.98	63.26	53.62	12.24	30.17	45.24	53.40	62.00	77.98
Mkt	0.64	0.03	0.58	0.62	0.63	0.65	0.69	0.86	0.03	0.80	0.84	0.86	0.88	0.93
HML	0.87	0.05	0.78	0.84	0.87	0.91	0.97	0.82	0.06	0.71	0.78	0.82	0.86	0.94
SMB	-0.15	0.06	-0.26	-0.19	-0.15	-0.11	-0.04	-0.71	0.06	-0.83	-0.75	-0.71	-0.67	-0.59
MOM	0.43	0.04	0.36	0.40	0.43	0.45	0.50	0.30	0.05	0.21	0.26	0.30	0.33	0.39
RMW	0.68	0.06	0.56	0.64	0.68	0.73	0.81	0.15	0.08	-0.01	0.10	0.15	0.21	0.32
CMA	0.26	0.06	0.13	0.21	0.26	0.30	0.38	0.33	0.08	0.17	0.27	0.33	0.38	0.48

Panel B. Subperiod 1: 1990 - 1998

	Updating protocol							Rolling protocol						
Coefficient	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
α	103.55	23.05	59.71	87.66	103.00	119.35	149.00	144.98	33.93	81.31	122.15	144.01	166.96	214.48
Mkt	0.20	0.07	0.06	0.15	0.20	0.24	0.33	0.14	0.11	-0.07	0.07	0.14	0.21	0.35
HML	1.68	0.18	1.34	1.56	1.68	1.80	2.05	2.29	0.25	1.82	2.12	2.28	2.45	2.79
SMB	0.50	0.11	0.28	0.42	0.50	0.58	0.73	0.15	0.15	-0.15	0.05	0.15	0.25	0.45
MOM	1.12	0.11	0.91	1.05	1.11	1.19	1.33	1.36	0.14	1.09	1.27	1.36	1.46	1.64
RMW	0.53	0.18	0.19	0.41	0.53	0.65	0.88	0.59	0.28	0.06	0.40	0.59	0.77	1.14
CMA	-0.63	0.22	-1.07	-0.78	-0.63	-0.48	-0.22	-1.29	0.32	-1.94	-1.50	-1.28	-1.07	-0.68

Panel C. Subperiod 2: 1999 - 2021

	Updating protocol							Rolling protocol						
Coefficient	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
α	4.75	8.28	-11.33	-0.82	4.79	10.36	21.21	-22.81	9.58	-41.57	-29.19	-22.89	-16.37	-3.78
Mkt	0.77	0.03	0.71	0.75	0.77	0.79	0.82	1.06	0.03	1.01	1.04	1.06	1.08	1.12
HML	0.67	0.05	0.58	0.64	0.67	0.70	0.76	0.47	0.05	0.37	0.43	0.47	0.50	0.57
SMB	-0.22	0.07	-0.36	-0.27	-0.22	-0.18	-0.09	-0.73	0.07	-0.87	-0.78	-0.73	-0.69	-0.60
MOM	0.37	0.04	0.30	0.35	0.37	0.40	0.45	0.20	0.05	0.10	0.16	0.20	0.23	0.29
RMW	0.84	0.06	0.72	0.80	0.84	0.88	0.97	0.39	0.08	0.24	0.34	0.39	0.44	0.55
CMA	0.38	0.07	0.25	0.33	0.38	0.42	0.51	-0.20	0.07	-0.34	-0.24	-0.19	-0.15	-0.06

Table IA-9

Out-of-Sample 6-factor Fama-French regressions

$$r_{i,t} - r_f = \alpha + \beta_1 \cdot (R_{m,t} - r_f) + \beta_2 \cdot \text{HML} + \beta_3 \cdot \text{SMB} + \beta_4 \cdot \text{MOM} + \beta_5 \cdot \text{RMW} + \beta_6 \cdot \text{CMA} + \epsilon_{i,t}$$

Full out-of-sample period: 1990 - 2021

For power utility investor with coefficient of relative risk aversion, $\gamma = 2$. Monthly returns; α in basis points per month.

	Updating Protocol							Rolling Protocol						
Coefficient	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
α	166.03	31.59	105.70	144.62	165.20	186.97	229.37	285.47	35.29	218.45	260.98	285.07	309.16	355.75
Mkt	0.62	0.09	0.45	0.56	0.62	0.68	0.79	0.89	0.09	0.72	0.83	0.89	0.95	1.07
HML	1.80	0.16	1.50	1.70	1.80	1.90	2.11	1.72	0.16	1.41	1.61	1.72	1.83	2.05
SMB	0.63	0.19	0.24	0.50	0.63	0.76	1.00	-1.28	0.22	-1.74	-1.42	-1.27	-1.13	-0.87
MOM	1.14	0.14	0.87	1.05	1.14	1.23	1.41	0.84	0.13	0.57	0.75	0.84	0.93	1.11
RMW	0.87	0.20	0.49	0.74	0.87	1.00	1.27	-0.62	0.30	-1.23	-0.83	-0.62	-0.42	-0.03
CMA	-0.32	0.22	-0.75	-0.47	-0.32	-0.17	0.12	-0.50	0.24	-0.97	-0.66	-0.50	-0.34	-0.03

Table IA-10

Out-of-Sample 6-factor Fama-French regressions
Subperiod 1: 1990 - 1998 Updating Protocol
Portfolio mean and variance decompositions

$$r_{i,t} - r_f = \alpha + \beta_1 \cdot (R_{m,t} - r_f) + \beta_2 \cdot \text{HML} + \beta_3 \cdot \text{SMB} + \beta_4 \cdot \text{MOM} + \beta_5 \cdot \text{RMW} + \beta_6 \cdot \text{CMA} + \epsilon_{i,t}$$

For power utility investor with coefficient of relative risk aversion, $\gamma = 8$. Monthly returns; α in basis points per month.

Panel A. Power utility investor with $\gamma = 5$

	Mean Return Decomposition							Variance Decomposition						
Coefficient	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
α / orthog.	42.21	6.52	28.64	38.00	42.45	46.75	54.38	52.83	4.02	45.03	50.14	52.79	55.51	60.72
Mkt	0.77	2.99	-5.29	-1.21	0.79	2.80	6.50	0.33	0.46	0.00	0.03	0.15	0.44	1.63
HML	4.28	1.82	10.93	13.03	14.20	15.47	18.06	43.77	7.79	29.18	38.34	43.55	48.89	59.85
SMB	-6.35	1.23	-8.91	-7.16	-6.31	-5.49	-4.09	10.13	3.34	4.41	7.71	9.88	12.28	17.42
MOM	43.58	4.57	35.22	40.42	43.41	46.54	53.06	32.48	4.35	24.07	29.55	32.46	35.44	41.00
RMW	7.87	3.07	2.05	5.79	7.81	9.89	14.09	1.57	1.07	0.10	0.77	1.37	2.17	4.16
CMA	-2.37	0.83	-4.04	-2.93	-2.36	-1.82	-0.78	4.69	2.88	0.49	2.58	4.24	6.34	11.48

Panel B. Power utility investor with $\gamma = 8$

	Mean Return Decomposition							Variance Decomposition						
Coefficient	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile	Mean	Std Dev	2.5%ile	25%ile	Median	75%ile	97.5%ile
α / orthog.	36.69	6.12	23.90	32.72	36.96	40.96	47.83	52.38	3.91	44.87	49.68	52.32	55.04	60.10
Mkt	7.14	2.68	1.96	5.31	7.10	8.91	12.51	2.16	1.54	0.13	1.03	1.86	2.95	6.02
HML	14.82	1.67	11.71	13.69	14.74	15.89	18.32	50.43	7.80	35.55	45.24	50.33	55.53	66.24
SMB	-4.61	1.12	-6.90	-5.35	-4.58	-3.84	-2.50	5.84	2.49	1.76	4.02	5.57	7.34	11.44
MOM	40.04	4.13	32.39	37.15	39.89	42.73	48.60	29.39	4.08	21.60	26.60	29.33	32.10	37.51
RMW	8.19	2.80	2.95	6.27	8.12	10.05	13.89	1.77	1.08	0.22	0.96	1.60	2.39	4.32
CMA	-2.26	0.77	-3.82	-2.77	-2.24	-1.74	-0.81	4.56	2.76	0.54	2.50	4.11	6.17	11.02